Computational Studies of Thermoelectric MHD in Molten Lithium

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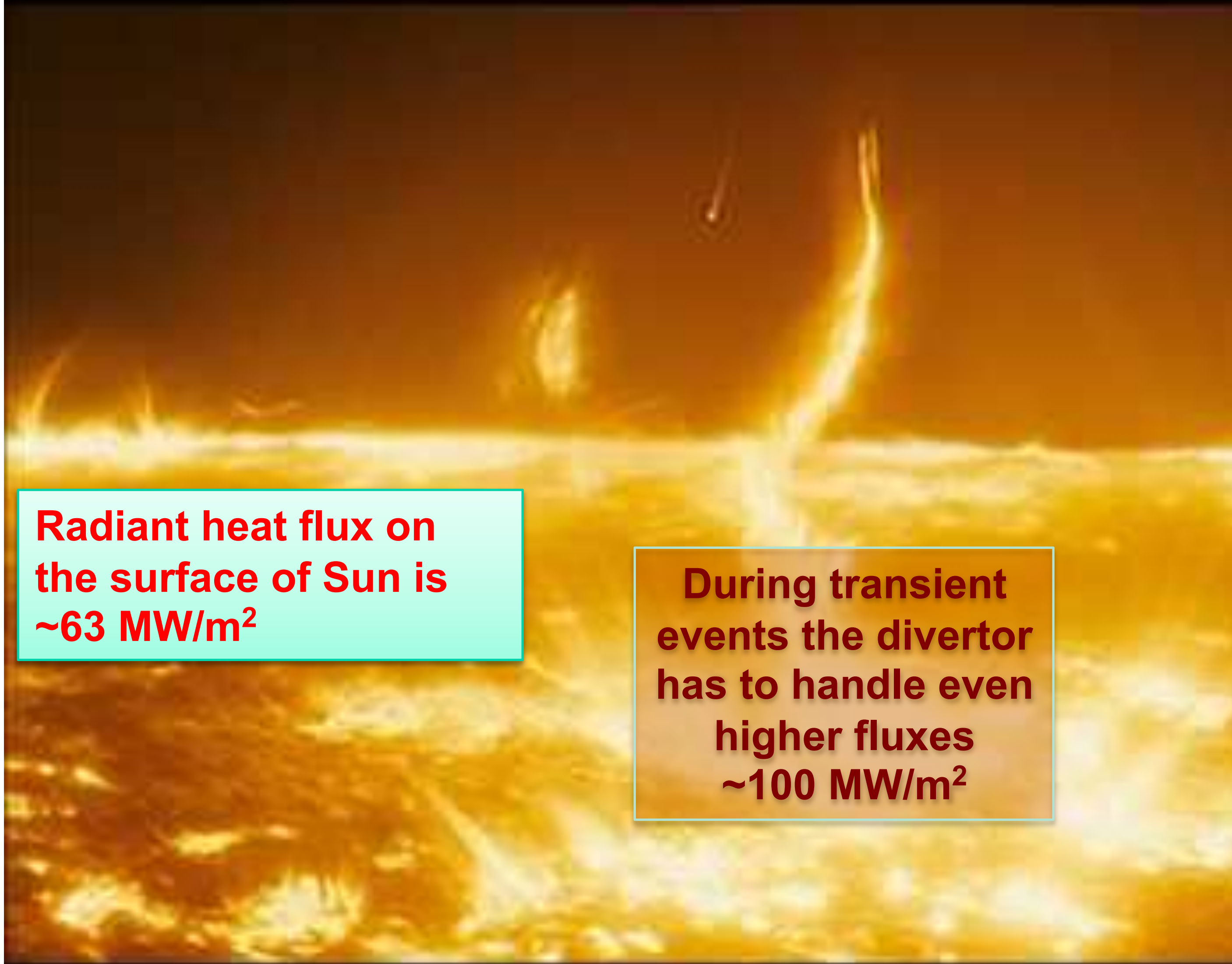
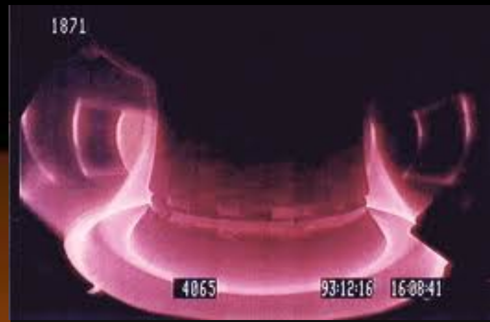
Outline

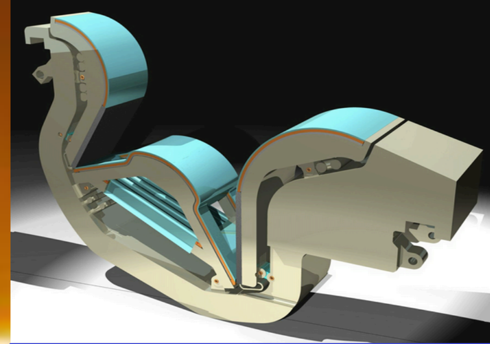
- Introduction
  - Why we're studying TEMHD of liquid lithium
  - Flowing liquid lithium for heat removal in tokamaks
  - Can we flow liquid lithium with TEMHD only?
  - What can we do with computations?
- Mathematical formulation of TEMHD
- Solution of TEMHD forces for several cases of interest
  - Two-metals junction
  - Stainless steel tray filled with lithium
  - Lithium inside an infinite stainless steel trench
  - Lithium inside a finite-size stainless steel trench
- Coupling with the velocity field, examples of solutions
- Conclusion



Radiant heat flux on the surface of Sun is ~63 MW/m^2

During transient events the divertor has to handle even higher fluxes ~100 MW/m^2





Heat flux on the divertor of a tokamak at steady-state is ~5-20 MW/m^2

Flowing liquid lithium for heat removal

In order to handle these gigantic heat fluxes, liquid lithium methods have been proposed for a continuous heat removal

- ✧ Pressure-driven (for example using a pump, SANDIA experim.)
- ✧ Gravity-assisted methods (falling film of lithium on a inclined surface, etc.)
- ✧ Surface-tension assisted (capillary and porous flows, FTU, TFTR)
- ✧ Thermoelectric-MHD assisted methods

Can we flow lithium using TEMHD forces only?

Can we exploit the high B-field of a tokamak to have a self-driven JxB flow of the molten lithium?

We're studying the possibility to drive liquid lithium on the divertor by using a self-driven TE-MHD flow

- **Temperature gradients** must be under careful control, since they govern the direction and magnitude of the TE force
- The **electrical boundary conditions** (like location of the electrical ground, charging of nearby materials, fluxes of electric charges from plasma, etc.) considerably affect the current path and must thus be known exactly
- The **sharing of electrical currents** between the plasma and the wall has to be predicted, for the management and containment of catastrophic events → This is an open issue

Issues

Main issues related to the use of lithium for ITER(*):

(1) Material compatibilities, long-term corrosion by lithium

(2) Tritium retention

(3) **Electromagnetic forces and their effects on the flowing liquid lithium divertor**

→ **Computational studies can aid on the evaluation of the electromagnetic forces before doing experimental tests**

(*) As raised by M. Shimada, *Conference Report on the 1st International Workshop on Li-applications to Boundary Control in Fusion Devices*, Nucl. Fusion 50 (2010) 077001, and *Conference Report on the 2nd International Workshop on Lithium Applications for Fusion Devices*, Nucl. Fusion 52 (2012) 037001

Thermoelectric Magnetohydrodynamics

$\vec{f}$

Thermoelectric problem

$\nabla^2 \phi = 0$ Electrostatics

$\nabla \cdot \vec{J} = 0$ Charge conservation

$\vec{J} = \sigma(-\nabla\phi - S\nabla T + \vec{u} \times \vec{B})$ Generalized Ohm Law

$C_p \rho \left[\frac{\partial T}{\partial t} + (\vec{u} \cdot \nabla) T \right] + \nabla \cdot q = \sigma^{-1} \vec{J} \cdot \vec{J}$ Diffusion advection of temperature

$q = -k\nabla T + ST\vec{J}$ Generalized Fourier Law

$\vec{f} = \vec{J} \times \vec{B} + \rho \vec{g}$ Specific force

Incompressible forced flow

$\nabla \cdot \vec{u} = 0$

$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} - \nu \nabla^2 \vec{u} + \frac{\nabla p}{\rho} = \frac{\vec{f}}{\rho}$

$\vec{u}$

Ref: J. A. Shercliff, J. Fluid Mech. 91, 2, 231-251 (1979)

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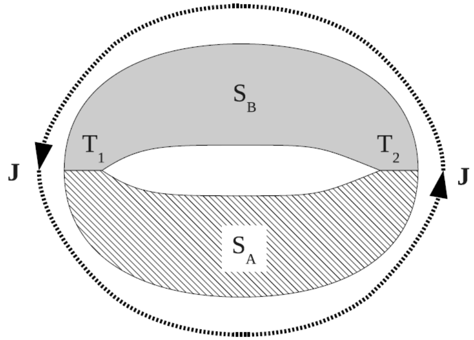
This problem is classical in fluid dynamics

$\vec{u}$

In this presentation we will spend most of the time on this problem, showing how the TE force develops for several cases of interest

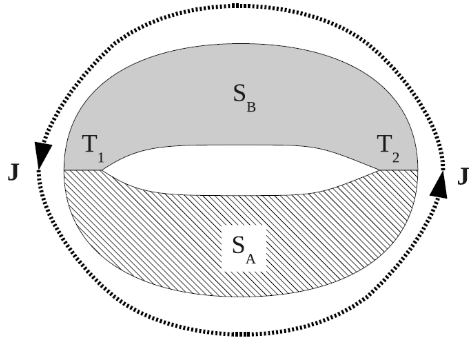
Ref: J. A. Shercliff, J. Fluid Mech. 91, 2, 231-251 (1979)

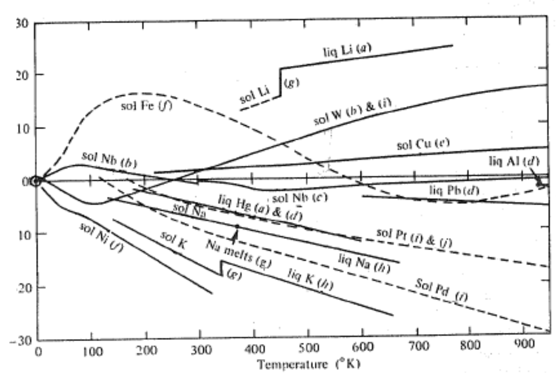
PROBLEM #1 : THERMOELECTRIC PROBLEM



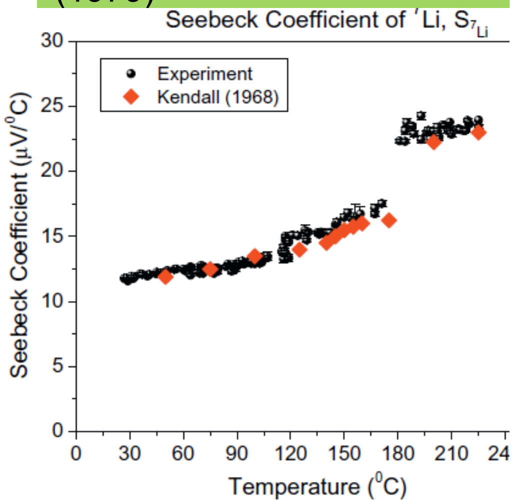
Thermoelectricity

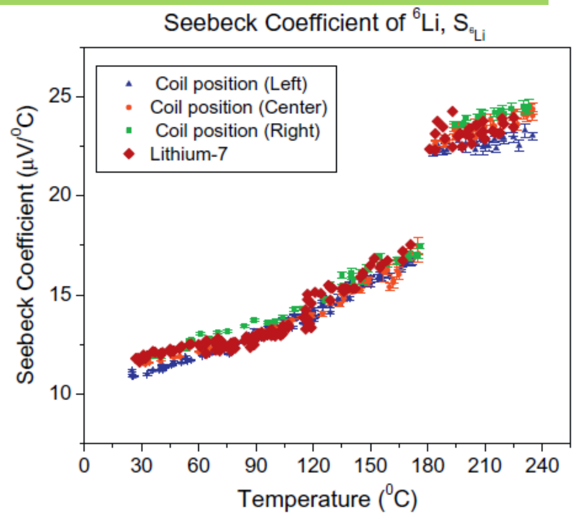
- Thermoelectric effect
  - Causes thermocouple junction voltage
  - Electric field generated by temperature gradient
  - Proportional to Seebeck coefficient (E=S*∂T/∂x)
  - Requires different material (or TE power) to provide current return path and to generate current
  - Lithium has a high Seebeck coefficient and is beneficial to fusion plasma. (low recycling, improved confinement, flat temperature profile and so on)





J.A. Shercliff, Thermoelectric MHD, J. Fluid Mech. **91**, 231 (1979)





V. Surla et. al., J. Nucl. Mater. 415, 18 (2011)

Seebeck coefficient measurements of lithium isotopes ☆

V. Surla [a,b,*], M. Tung [a], W. Xu [a], D. Andruczyk [a], M. Neumann [a], D.N. Ruzic [a], D. Mansfield [b]

[a] Center for Plasma–Material Interactions, Department of Nuclear, Plasma and Radiological Engineering, University of Illinois at Urbana Champaign, Urbana, IL 61801, USA
[b] Princeton Plasma Physics Laboratory, Princeton, NJ 08543, USA

Governing equations

The minimal set to see Thermoelectricity in action

$$J = -\sigma\nabla\phi - \sigma S\nabla T$$

Generalized Ohm

$$q = -k\nabla T$$

Fourier Law

$$\nabla \cdot J = 0$$

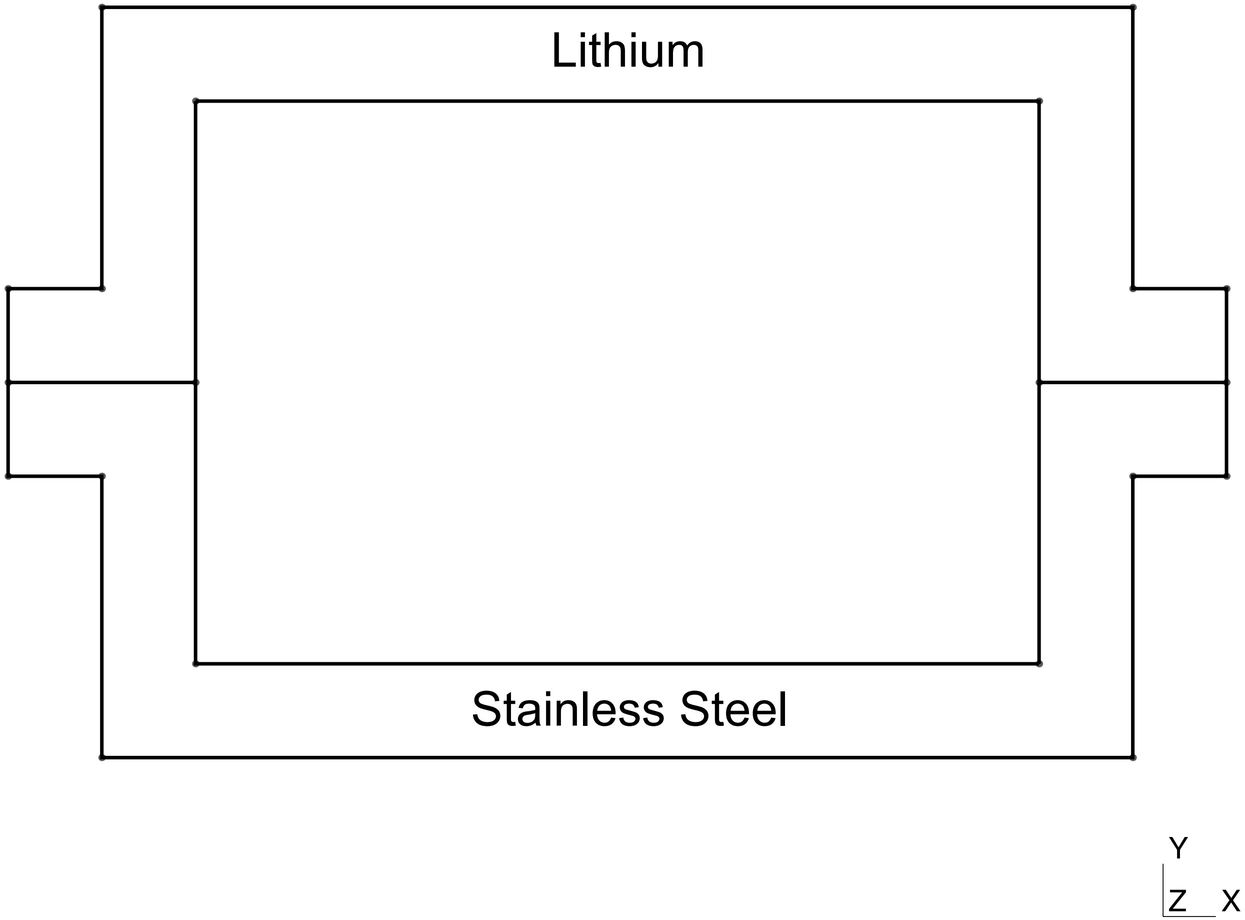
Current Continuity

$$\nabla \cdot q = 0$$

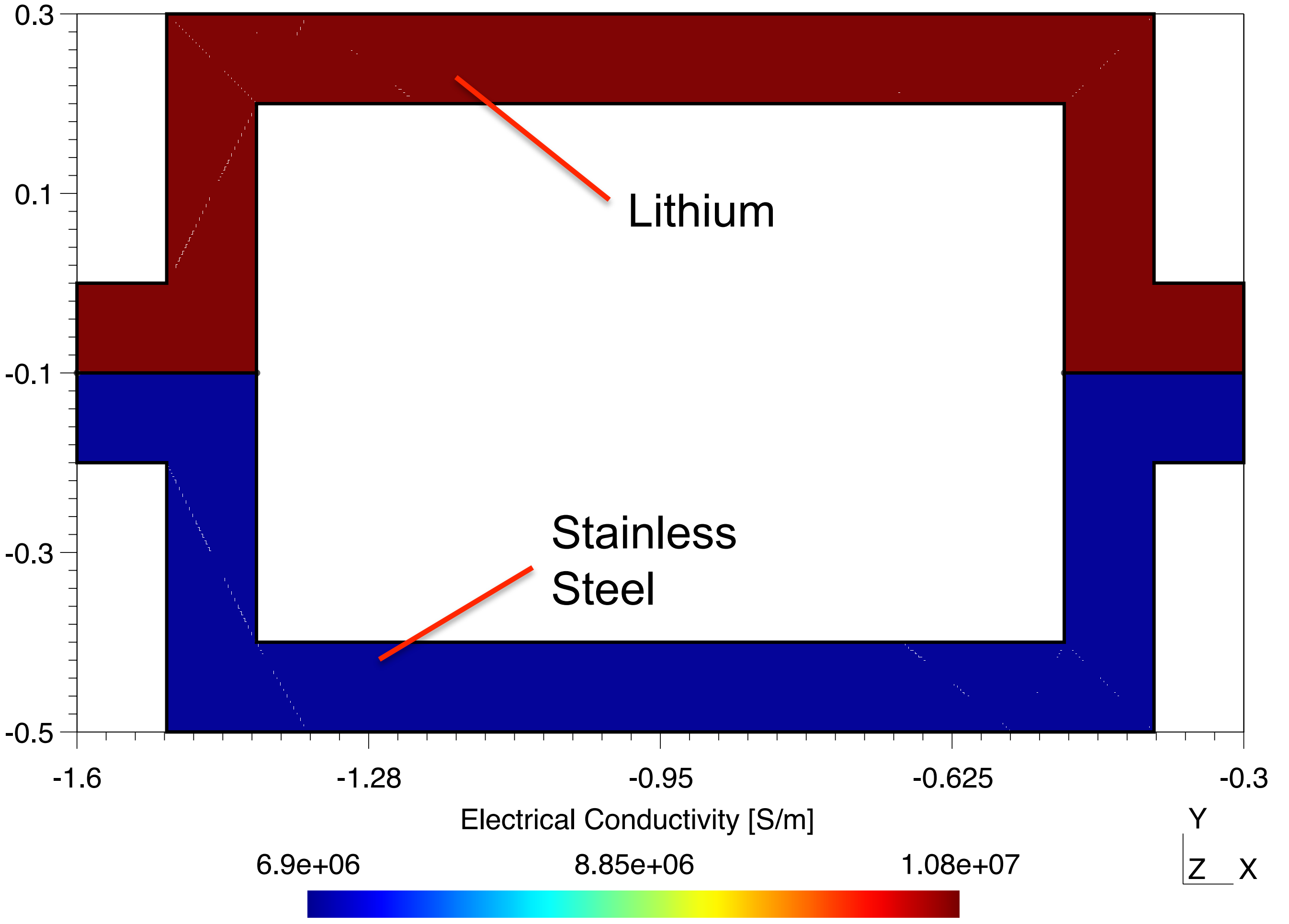
Simplified Heat Balance

Thermoelectricity of a solid junction

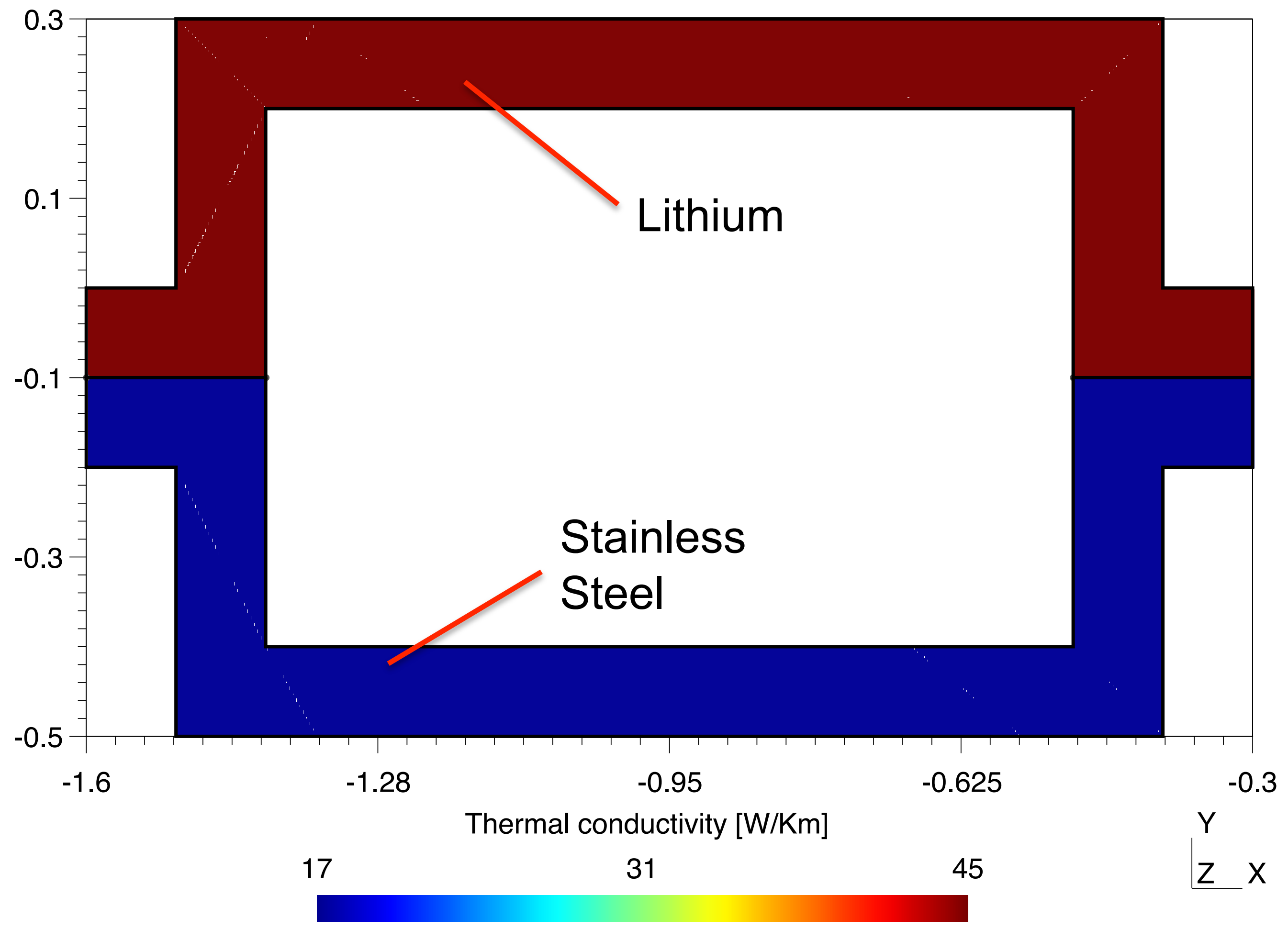
Let's think about the classical 2-metal junction, let's keep Li&SS



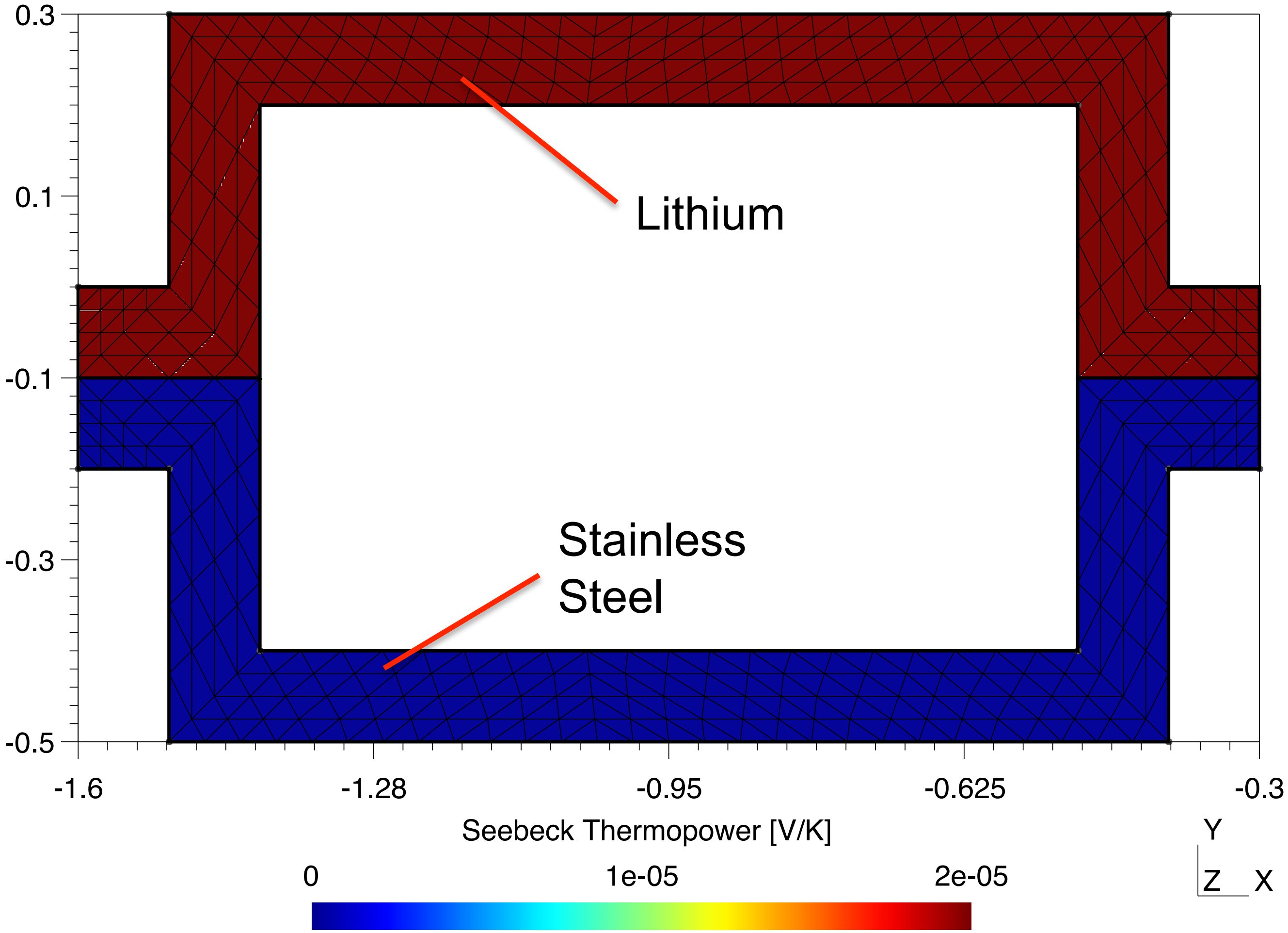
Thermoelectricity of a solid junction



Thermoelectricity of a solid junction



Thermoelectricity of a solid junction

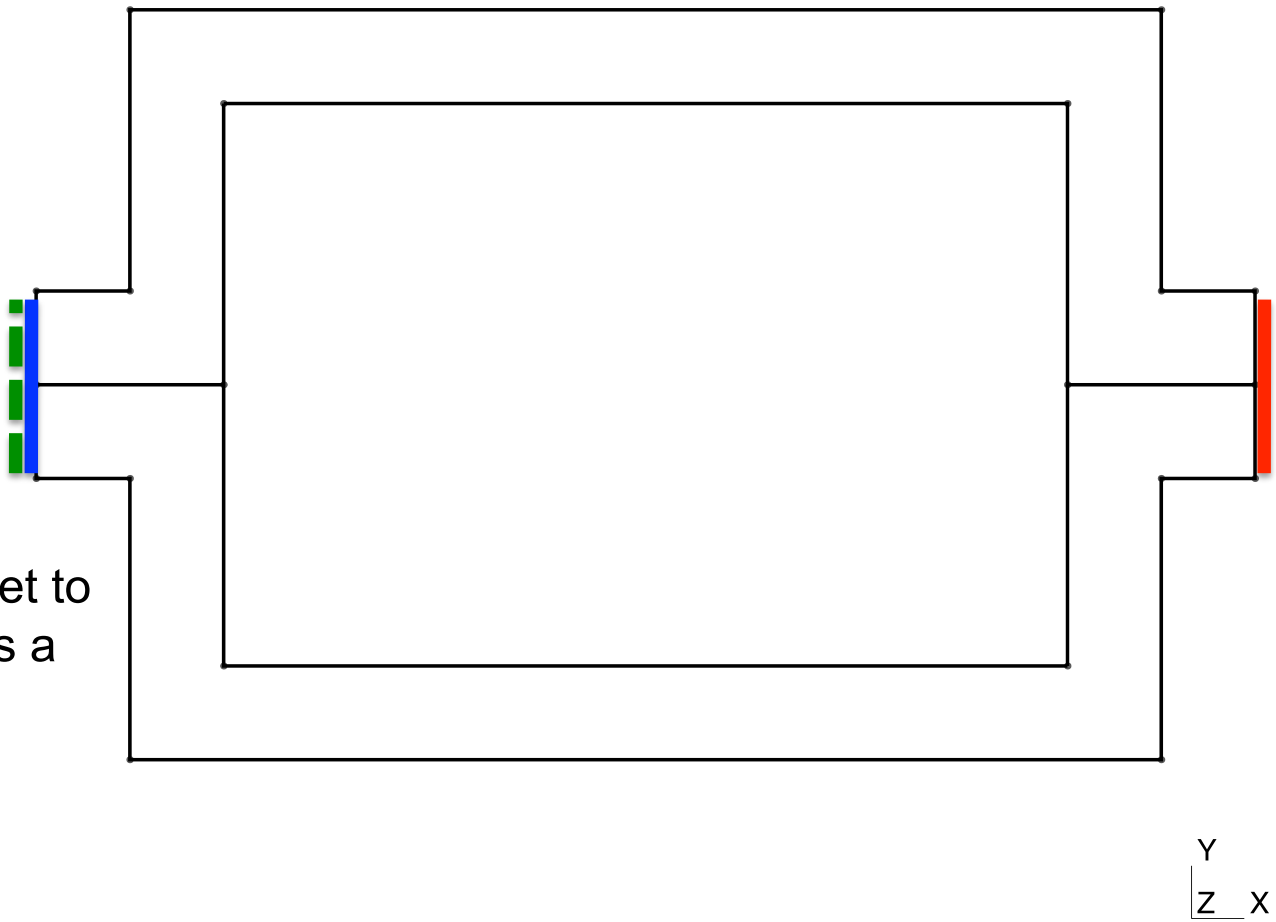


Thermoelectricity of a solid junction

Boundary Conditions

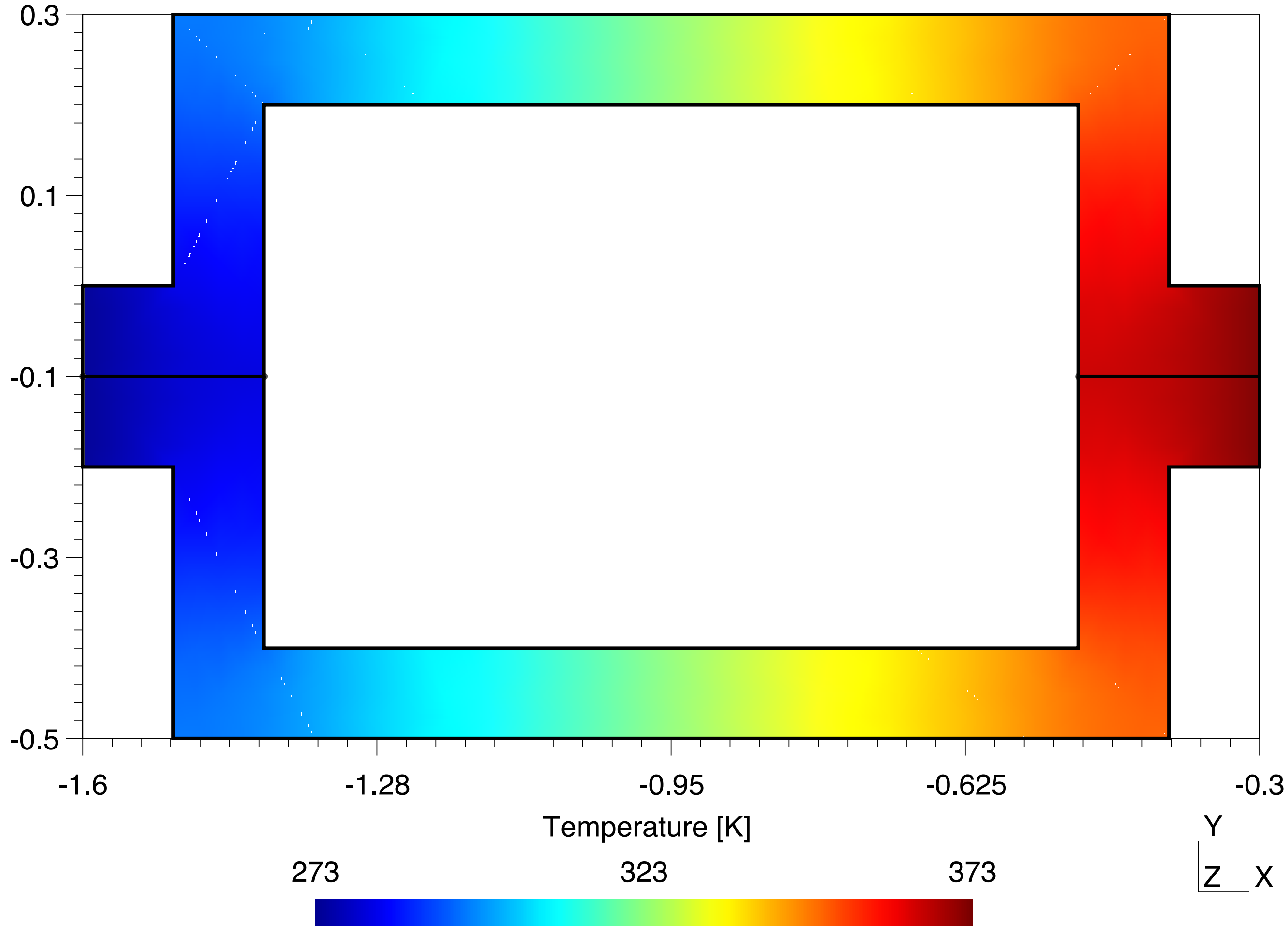
Cold Side
T = 273K,

electric potential set to 0.0 Volt, as a reference

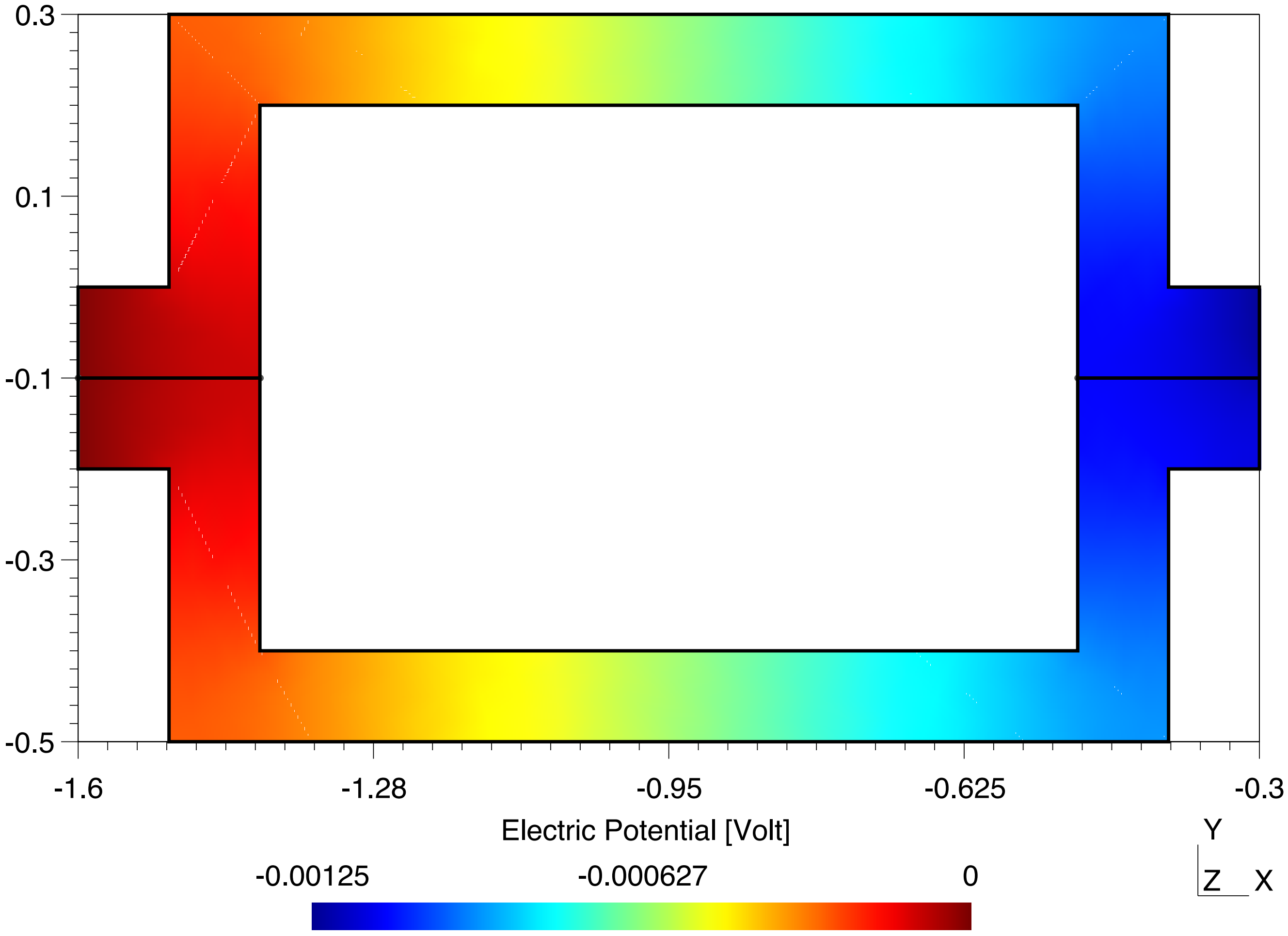


Hot Side
T = 373K

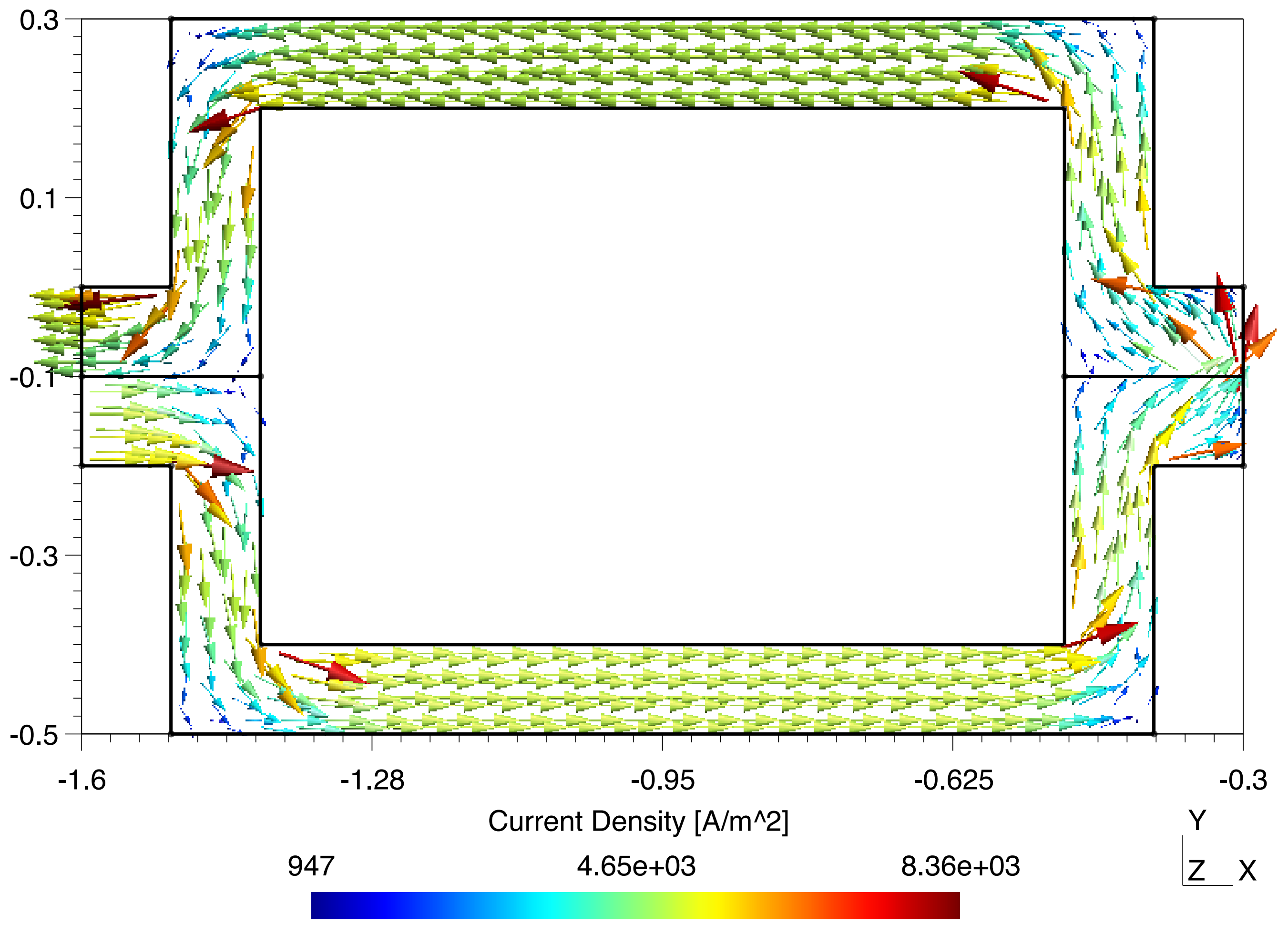
Thermoelectricity of a solid junction



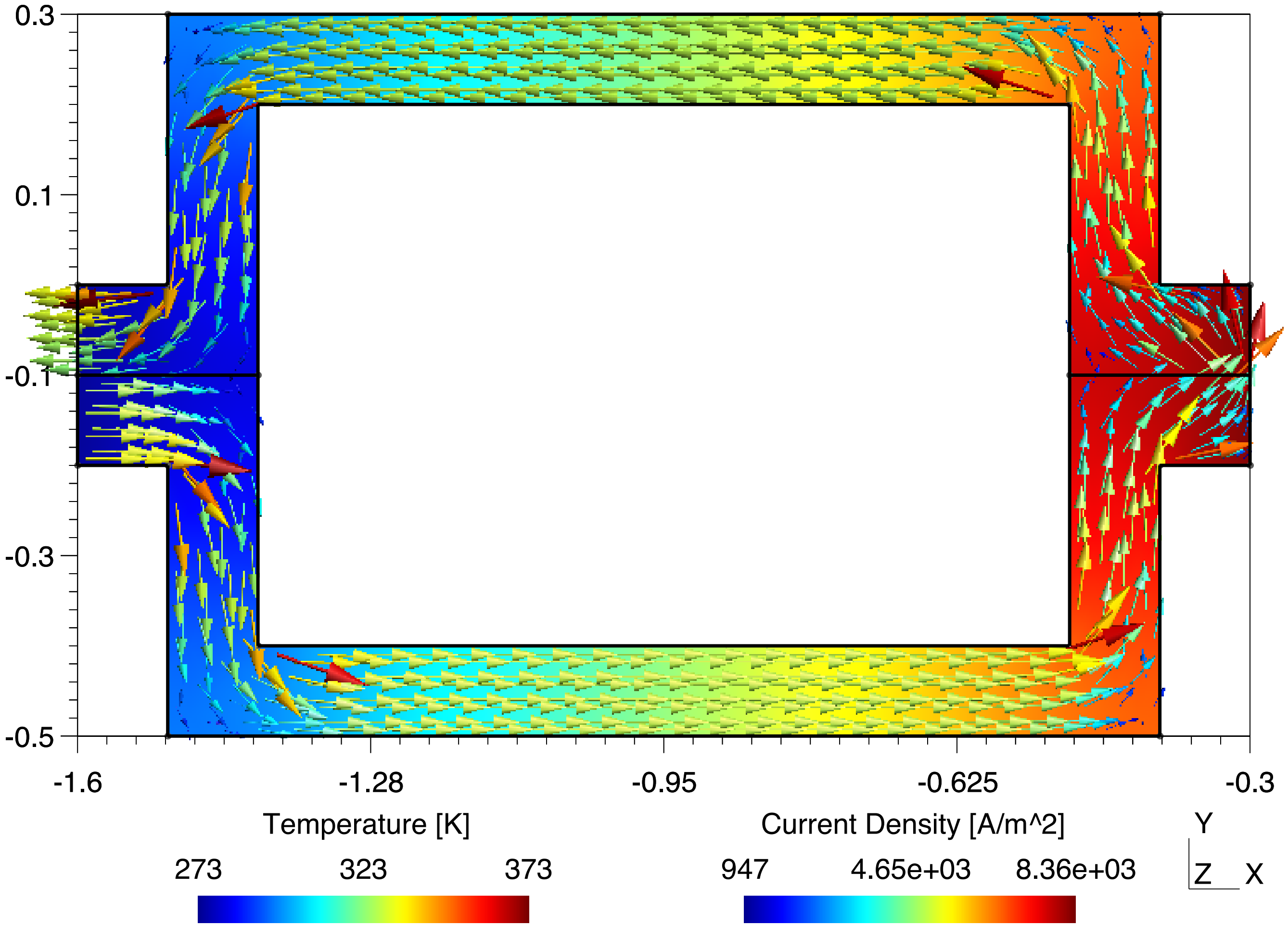
Thermoelectricity of a solid junction



Thermoelectricity of a solid junction

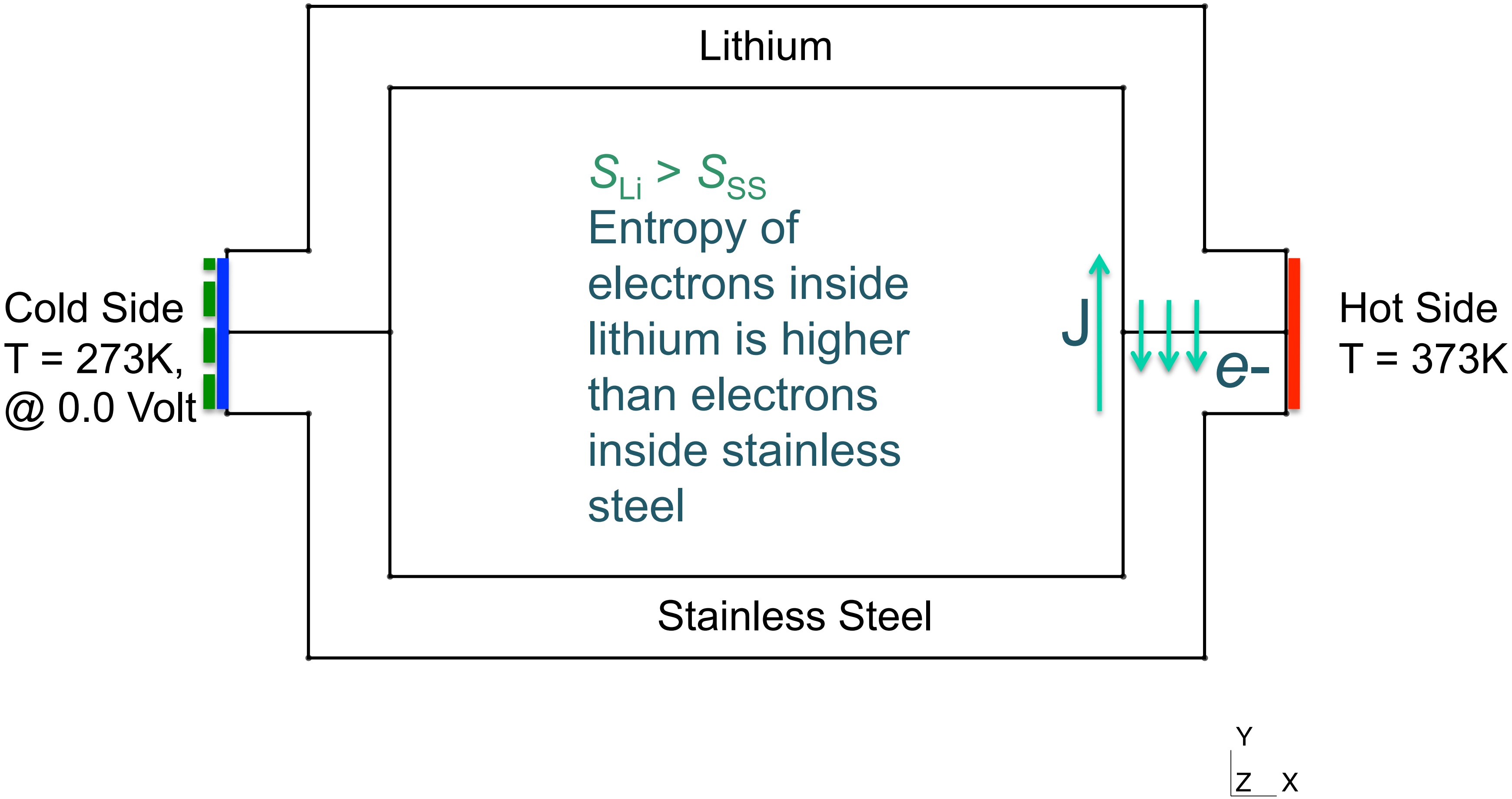


Thermoelectricity of a solid junction



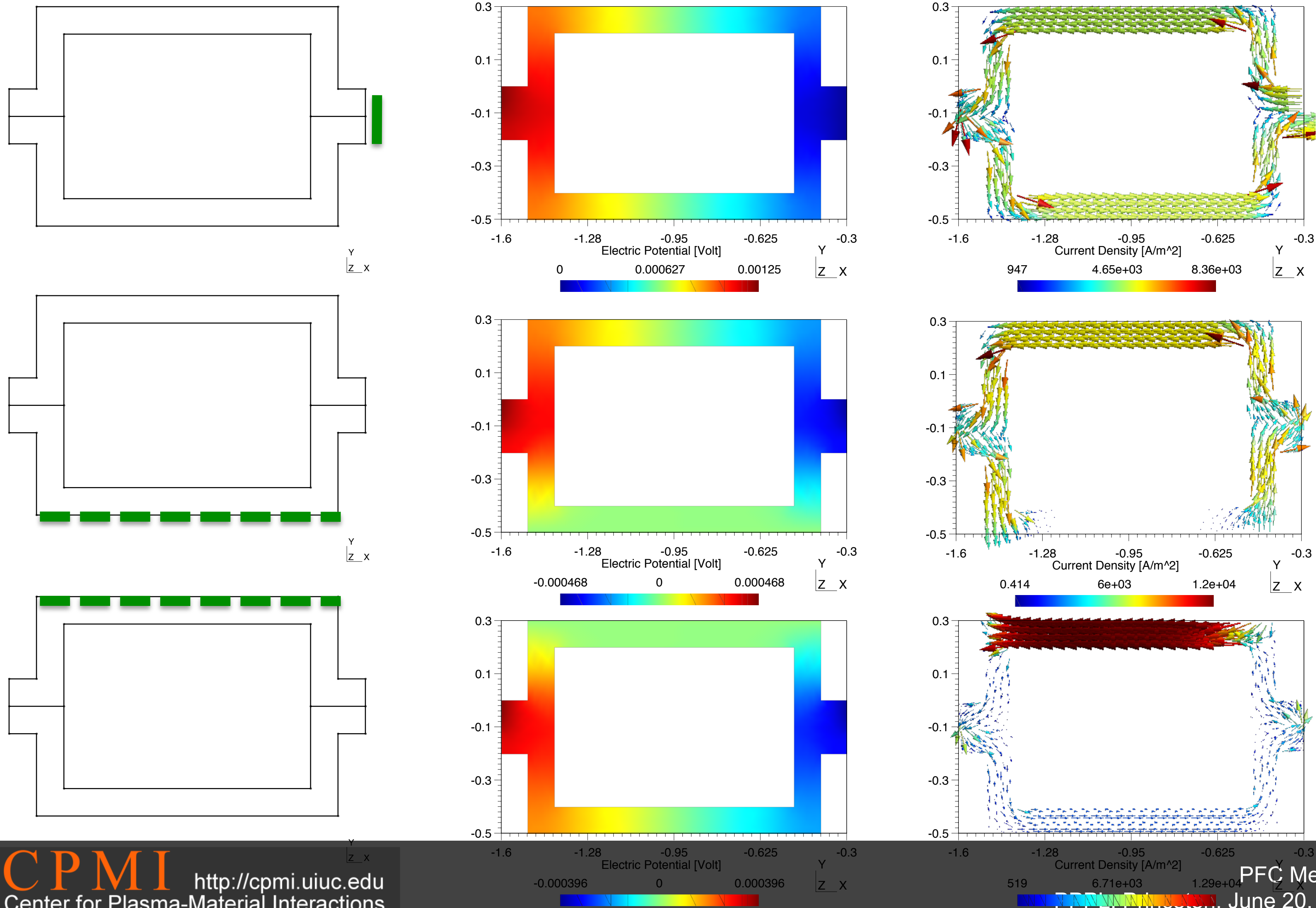
Thermoelectricity of a solid junction

Meaning of Thermopower *S*



Thermoelectricity of a solid junction

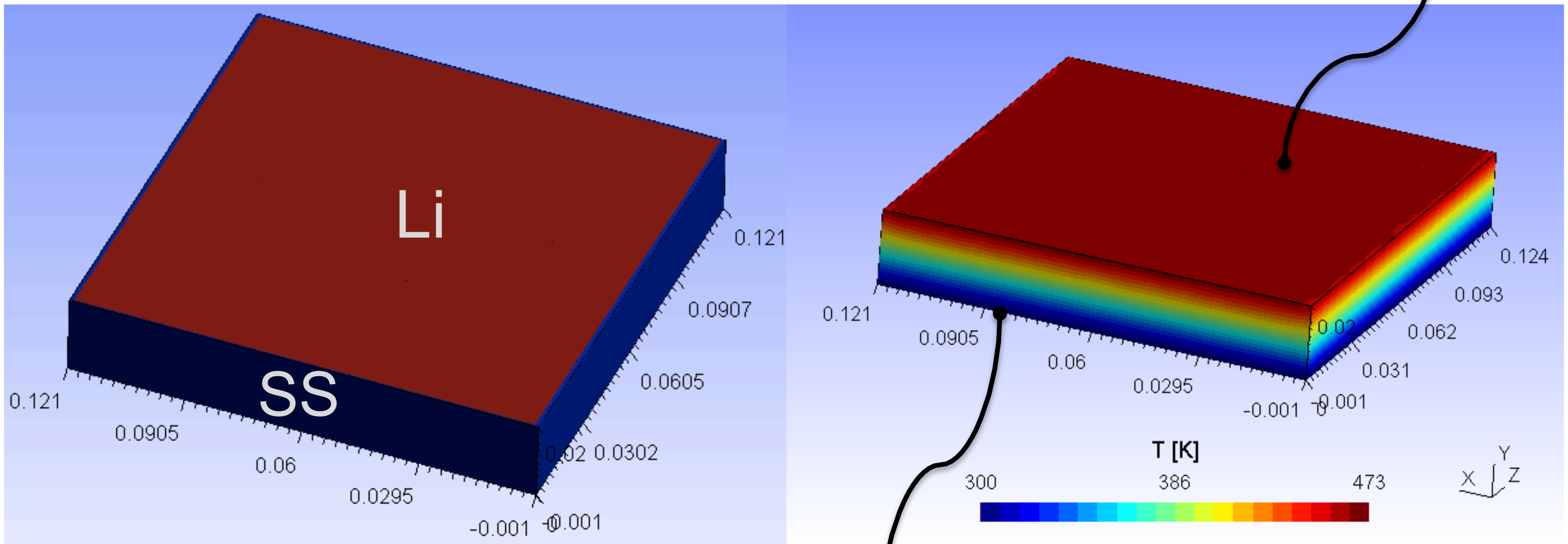
What happens if we change the location of the ground?



Stainless steel tray filled with lithium

The thermoelectricity of a SS tray filled with lithium works similarly to a thermocouple junction

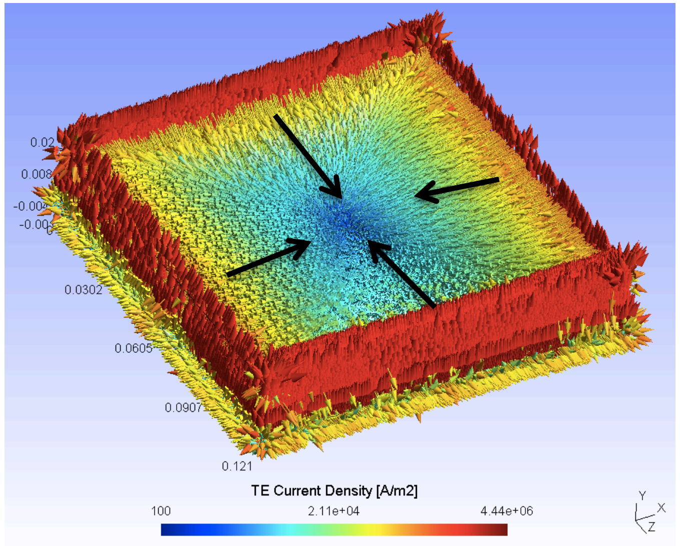
Top face heated uniformly

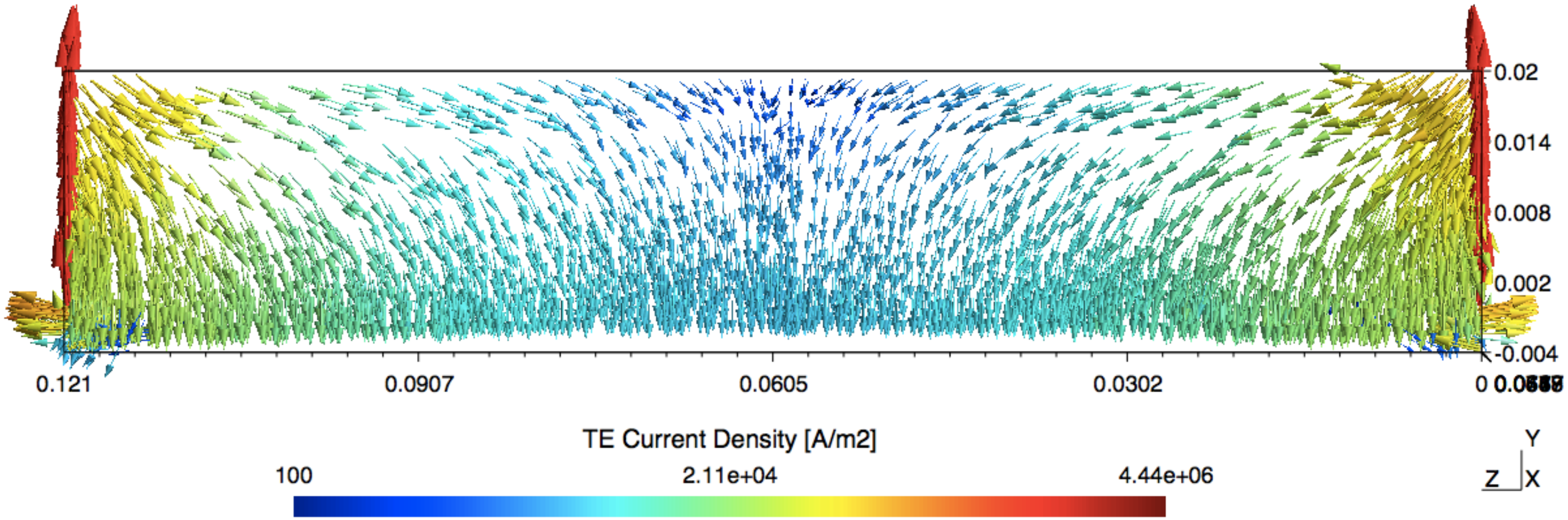


Bottom is cooled and electrically grounded

Stainless steel tray filled with lithium

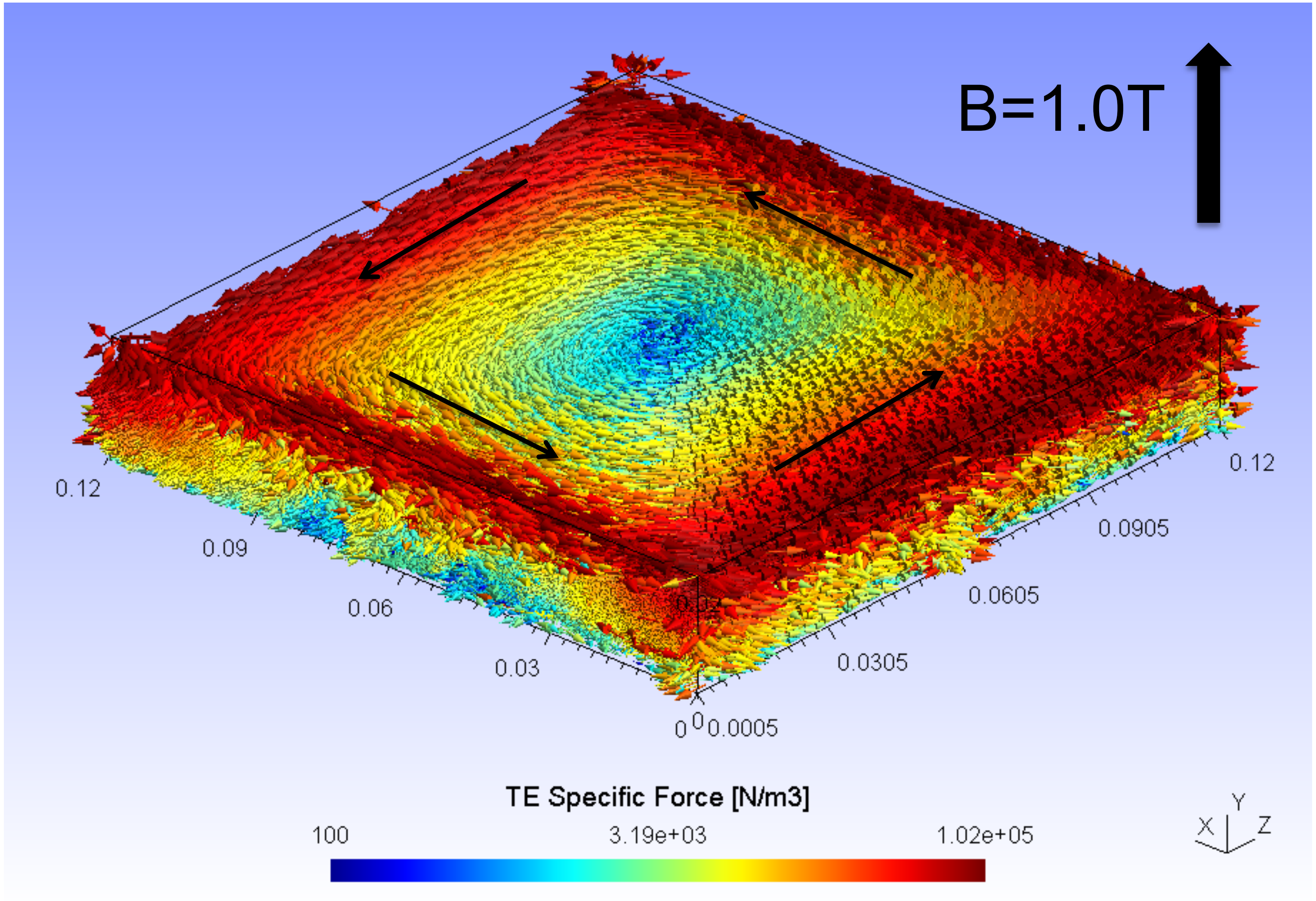
- Current Density
  - The TE electrical current is generated at the interface between the two materials
  - The current density vectors point inward, sinking down at the center of the tray
  - The vectors falls perpendicular to the bottom of the tray, which is grounded





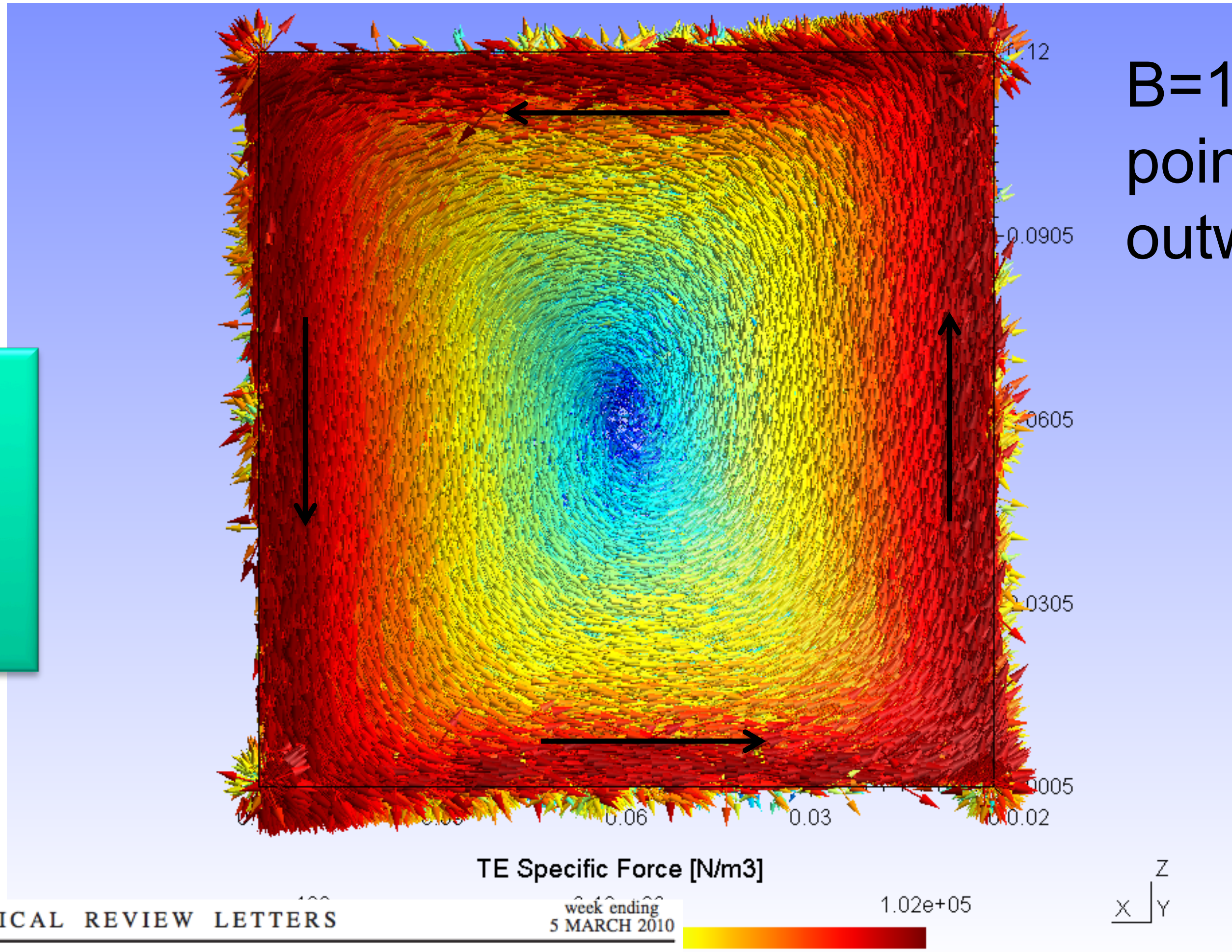
Stainless steel tray filled with lithium

Thermoelectric JxB Specific Force acting on the lithium



Stainless steel tray filled with lithium

We can circulate liquid lithium inside the tray!



B=1.0T, pointing outward

PRL **104,** 094503 (2010) PHYSICAL REVIEW LETTERS week ending 5 MARCH 2010

Thermoelectric Magnetohydrodynamic Stirring of Liquid Metals

M. A. Jaworski,* T. K. Gray, M. Antonelli, J. J. Kim, C. Y. Lau, M. B. Lee, M. J. Neumann, W. Xu, and D. N. Ruzic
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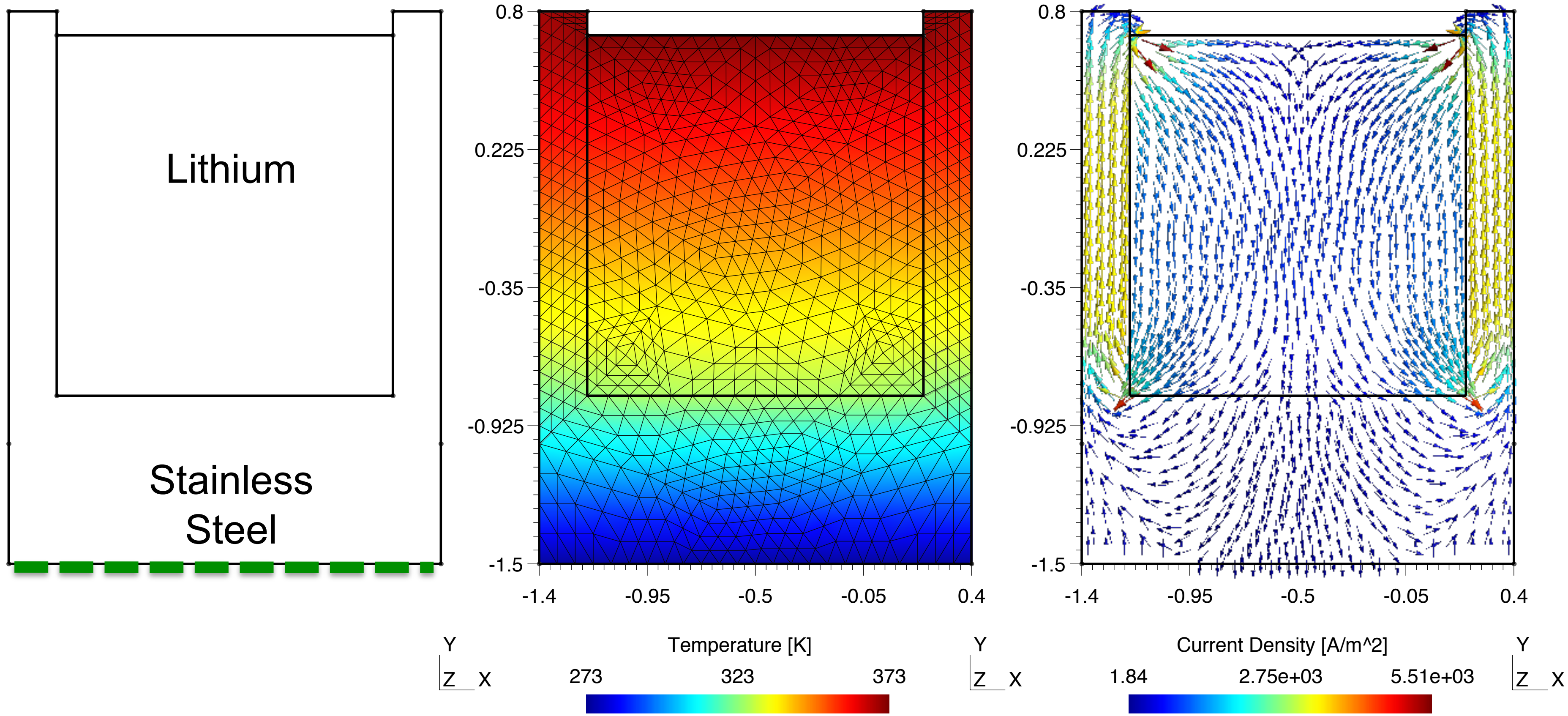
(Received 10 December 2009; revised manuscript received 25 January 2010; published 5 March 2010)

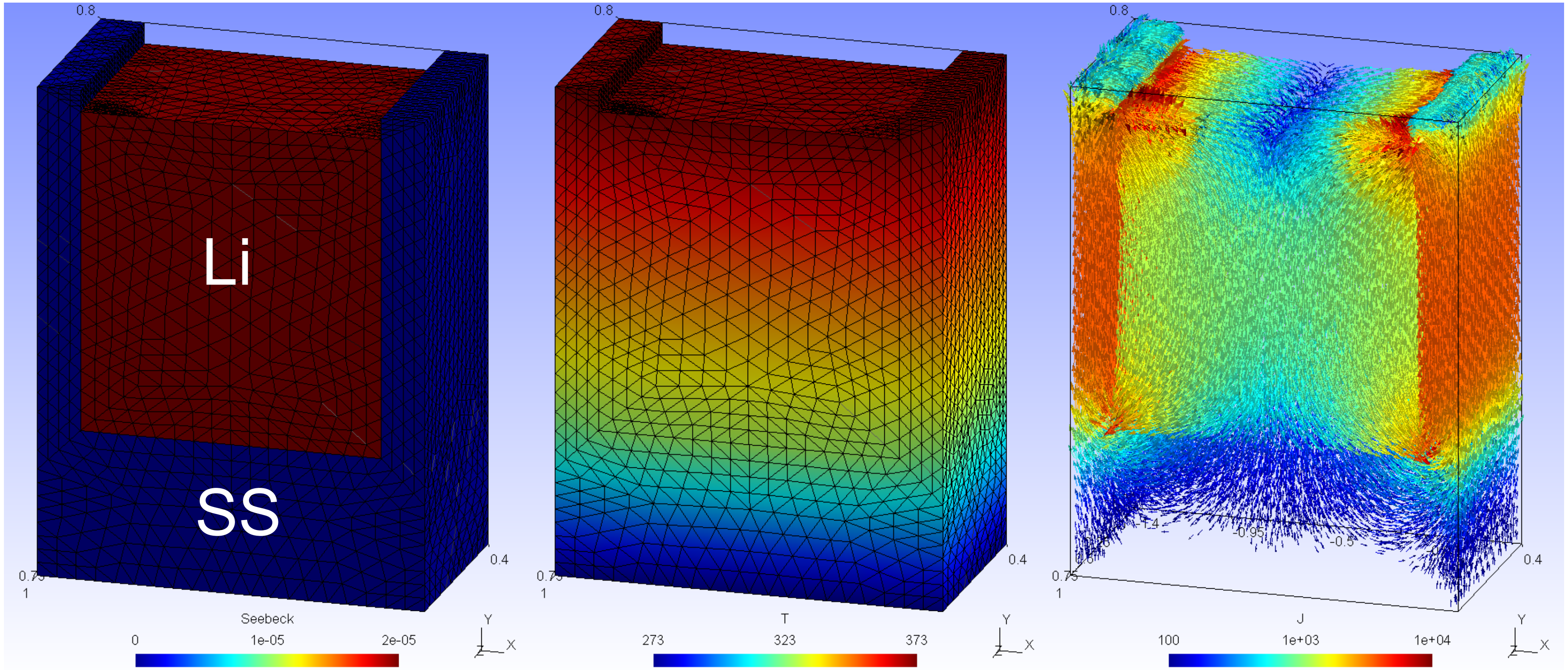
Lithium inside a stainless steel trench

- We consider a SS trench containing the Lithium
  - Heated on top, grounded on bottom
  - The thermoelectric currents are generated at the Li-SS interface



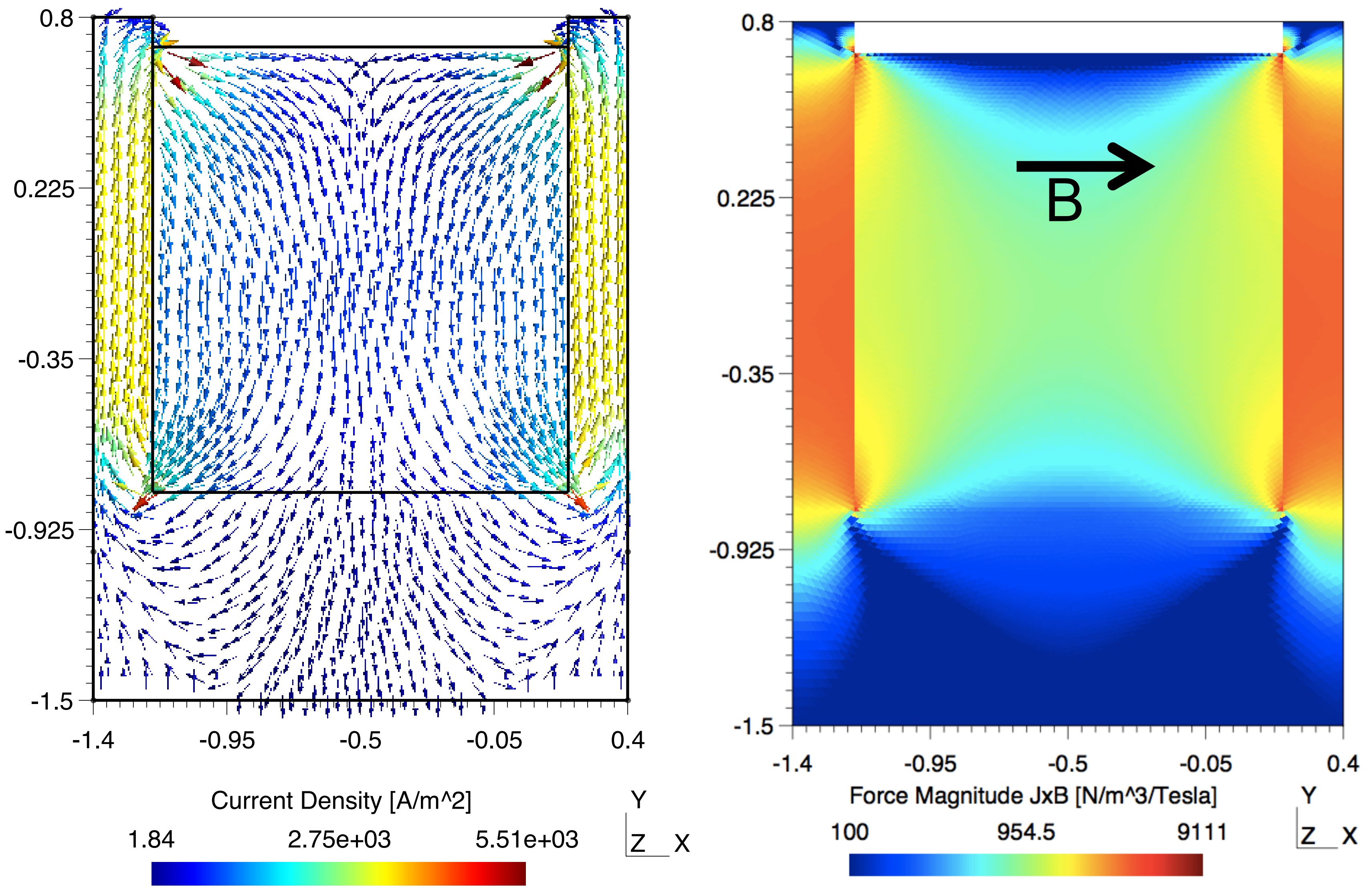
Lithium inside a stainless steel trench

- 3D picture, infinite trench



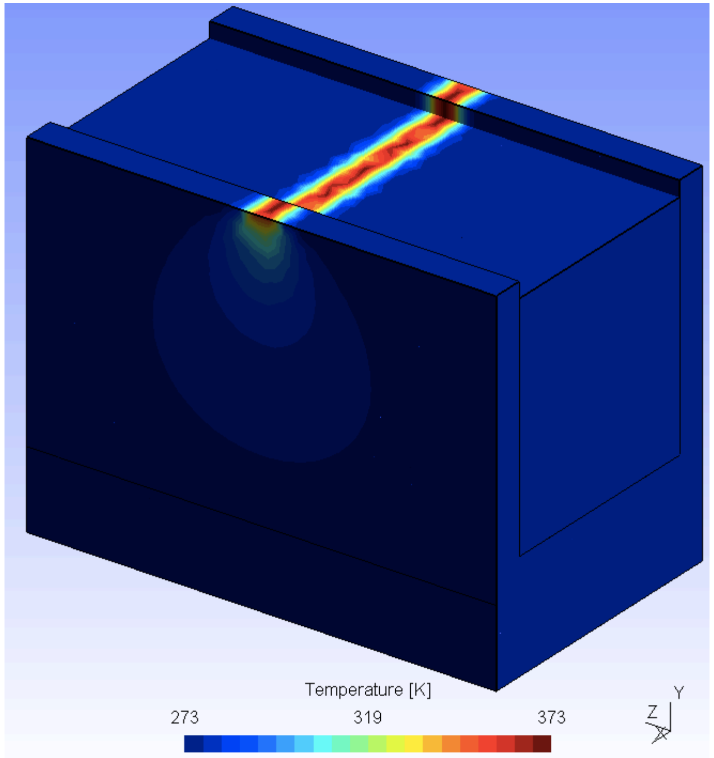
Temperature   Current Density

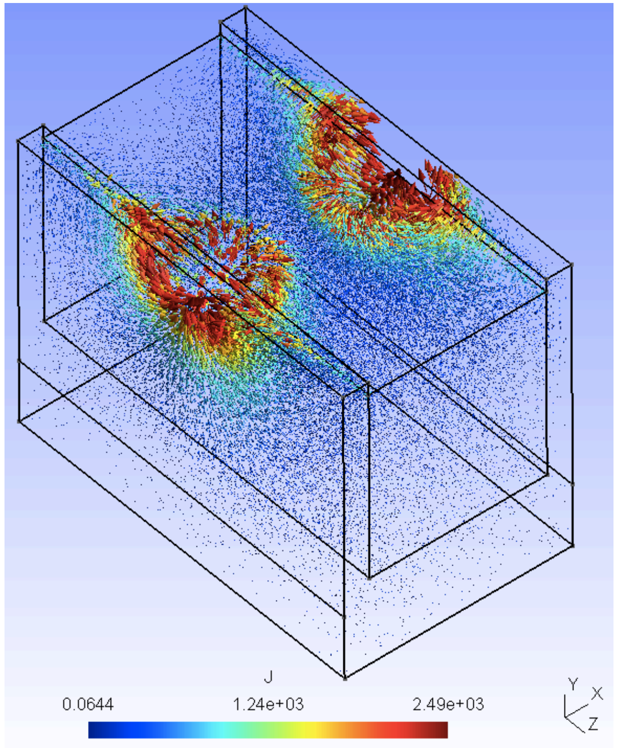
Lithium inside a stainless steel trench



Lithium inside a stainless steel trench

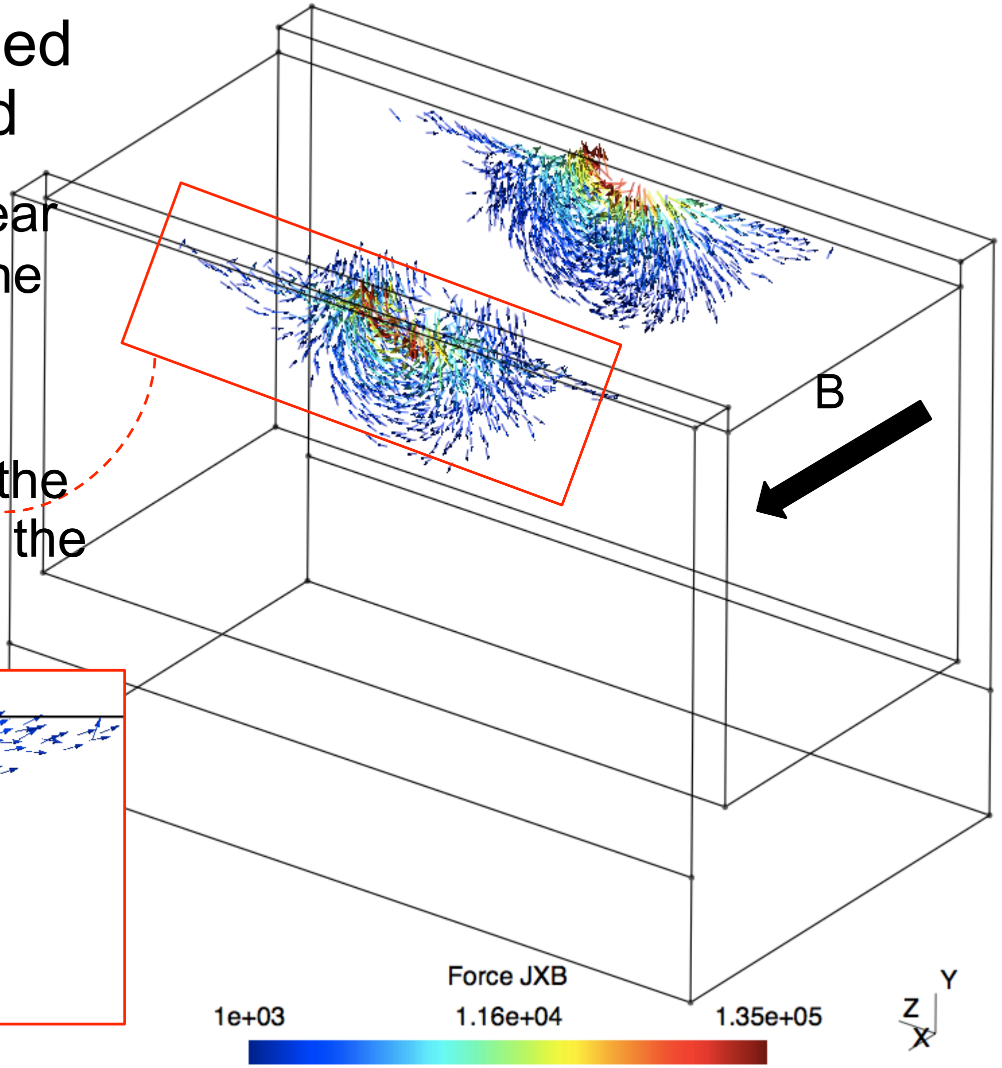
- In case of a localized heating, like a **Gaussian heating at center** along the transverse direction of the infinite trench, more complex current path are developed
- TE currents are developed at the Li-SS interface near the heated region

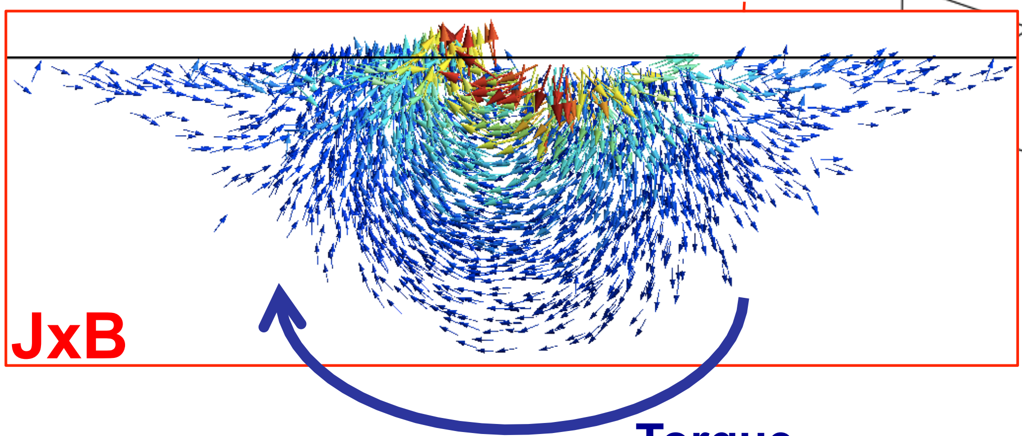




Lithium inside a stainless steel trench

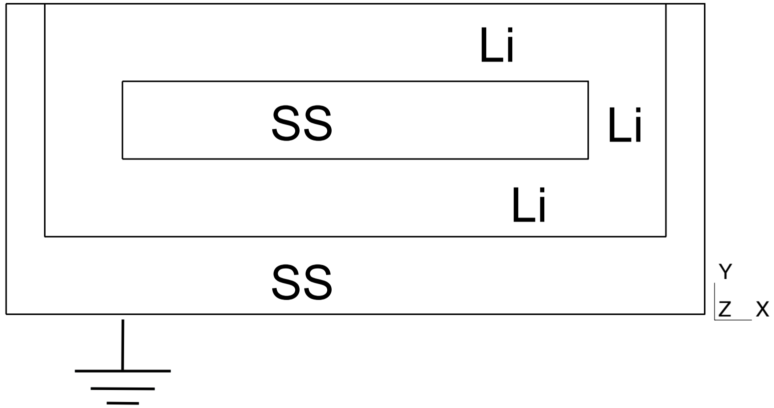
- Thermoelectric force developed in case of a transversal **B** field
  - Most of the TE force develops near the two-metal interfaces, where the TE currents have the highest intensity
  - The TE force exerts a **torque** on the lithium centred around the axis of the heated metal junction

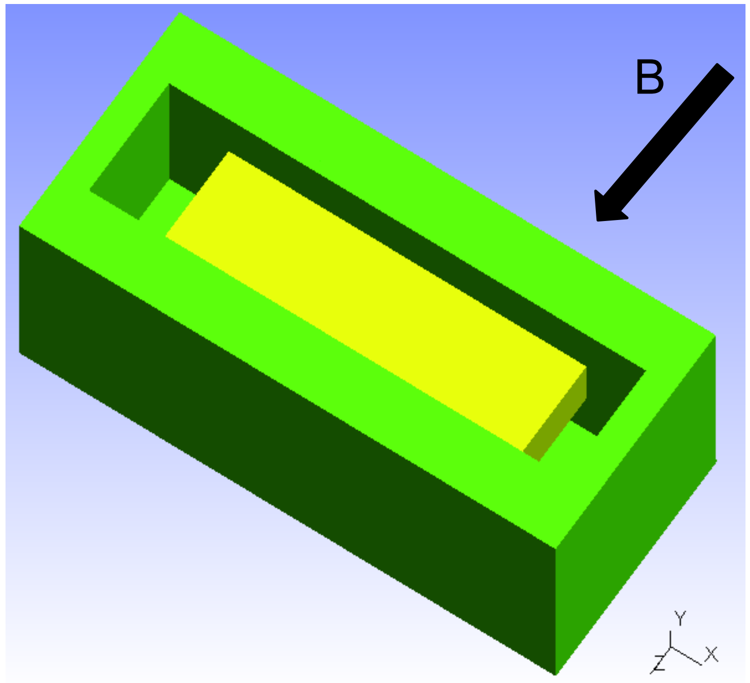




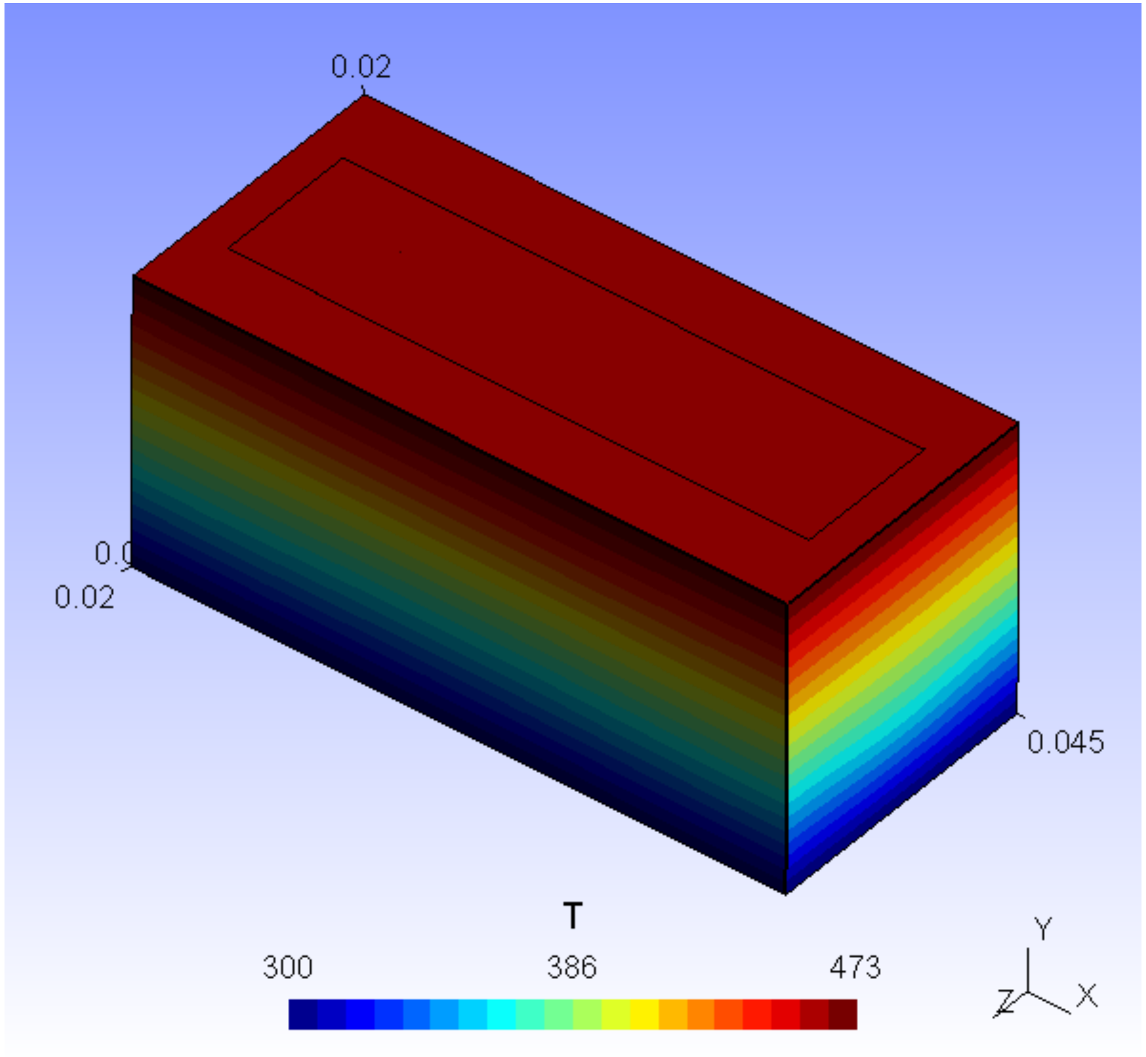
Lithium circulation with TE-MHD pumping

- In order to assess the feasibility of a pumping system for liquid lithium using TE-MHD only, we consider a 3D model of a finite-size stainless steel trench of 2.0 x 2.0 x 4.5 cm (green) comprising a SS separation plate at center (yellow)
- Liquid lithium fills the cavity up to the top
- The bottom is electrically grounded
- All the other walls are electrically insulated
- A magnetic field B is directed transversally
- The feasibility to drive liquid lithium using TE-MHD forces in this kind of system depends on the temperature gradient established





HEATING SCENARIO #1



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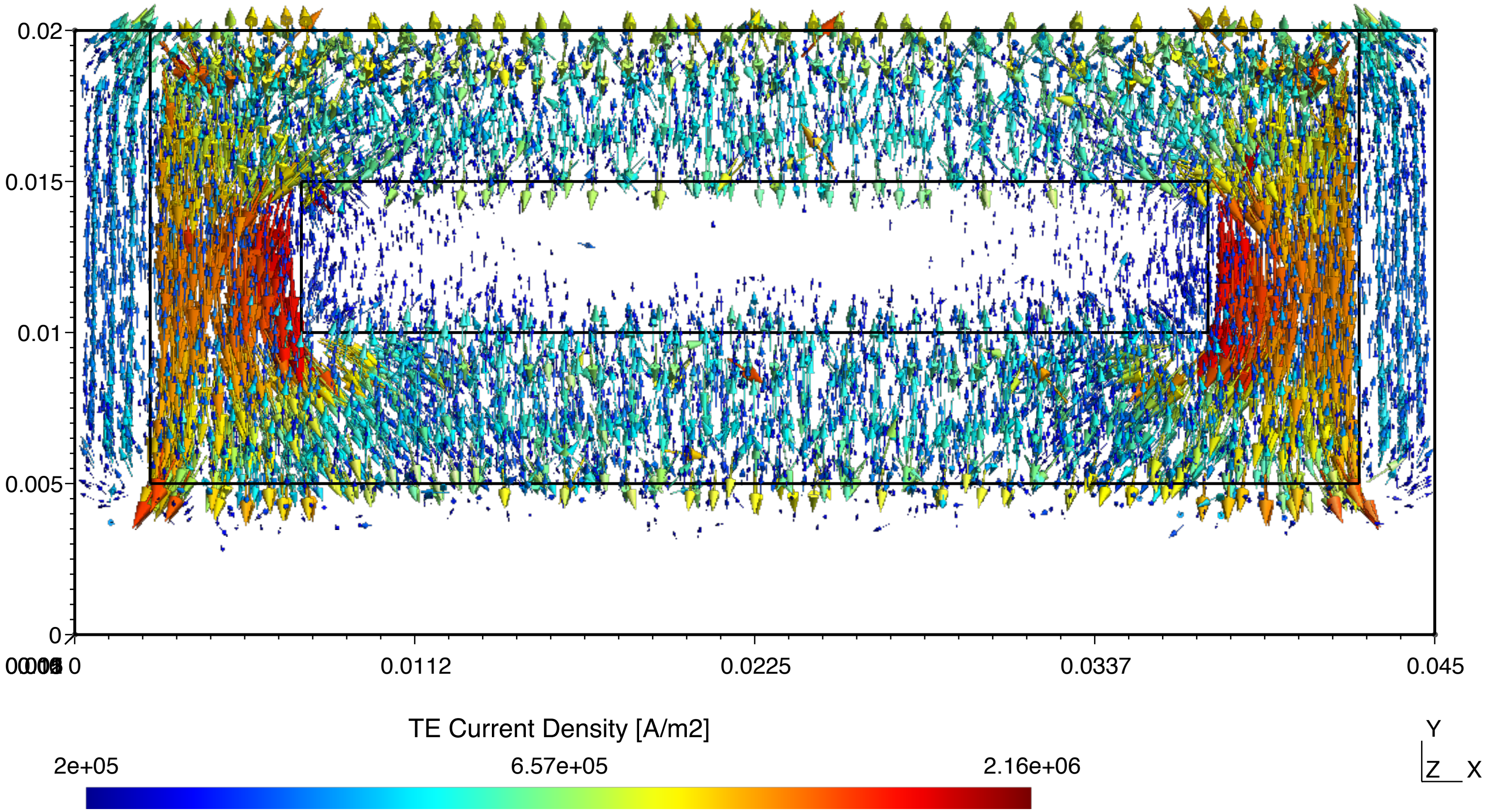

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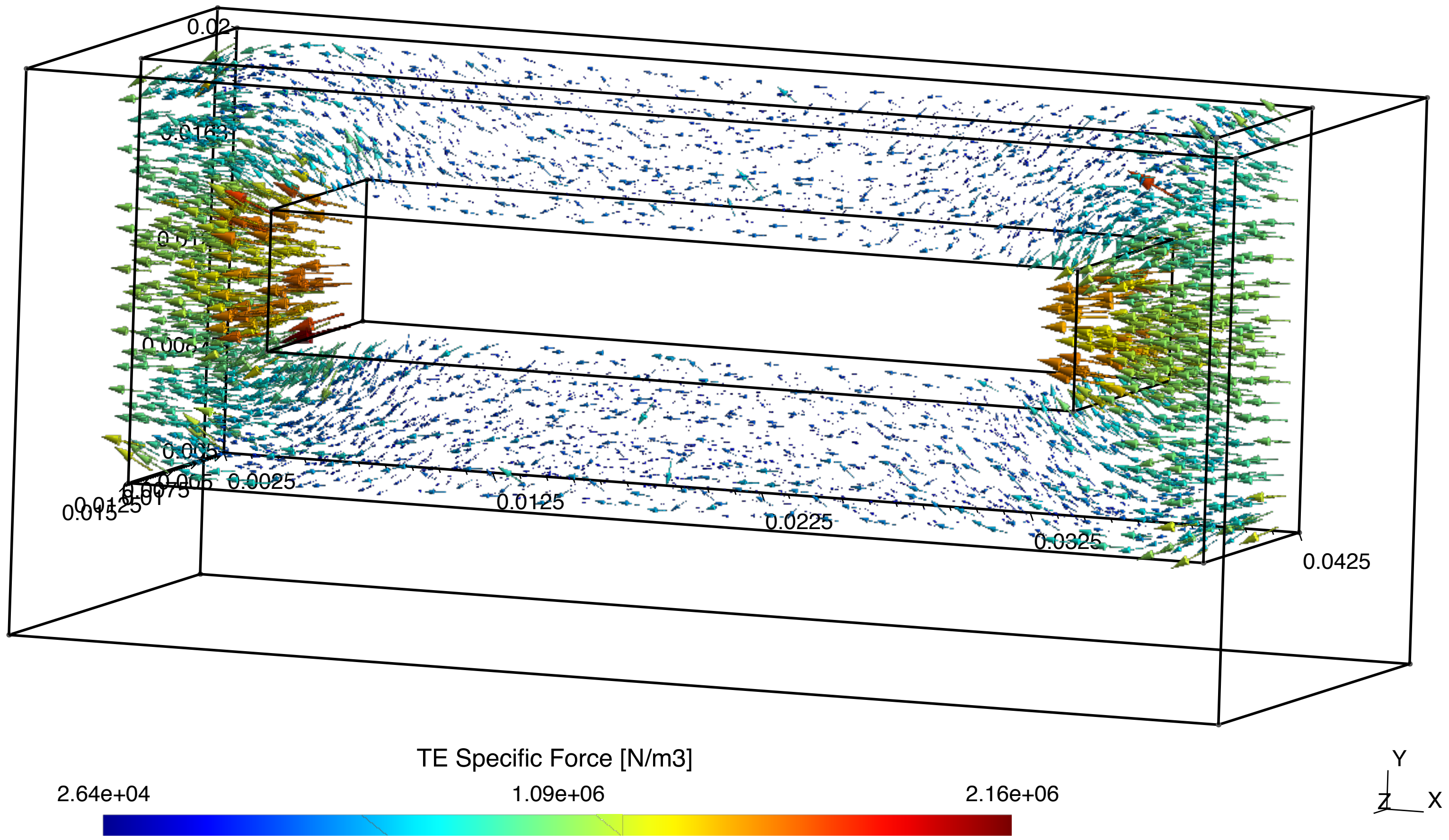

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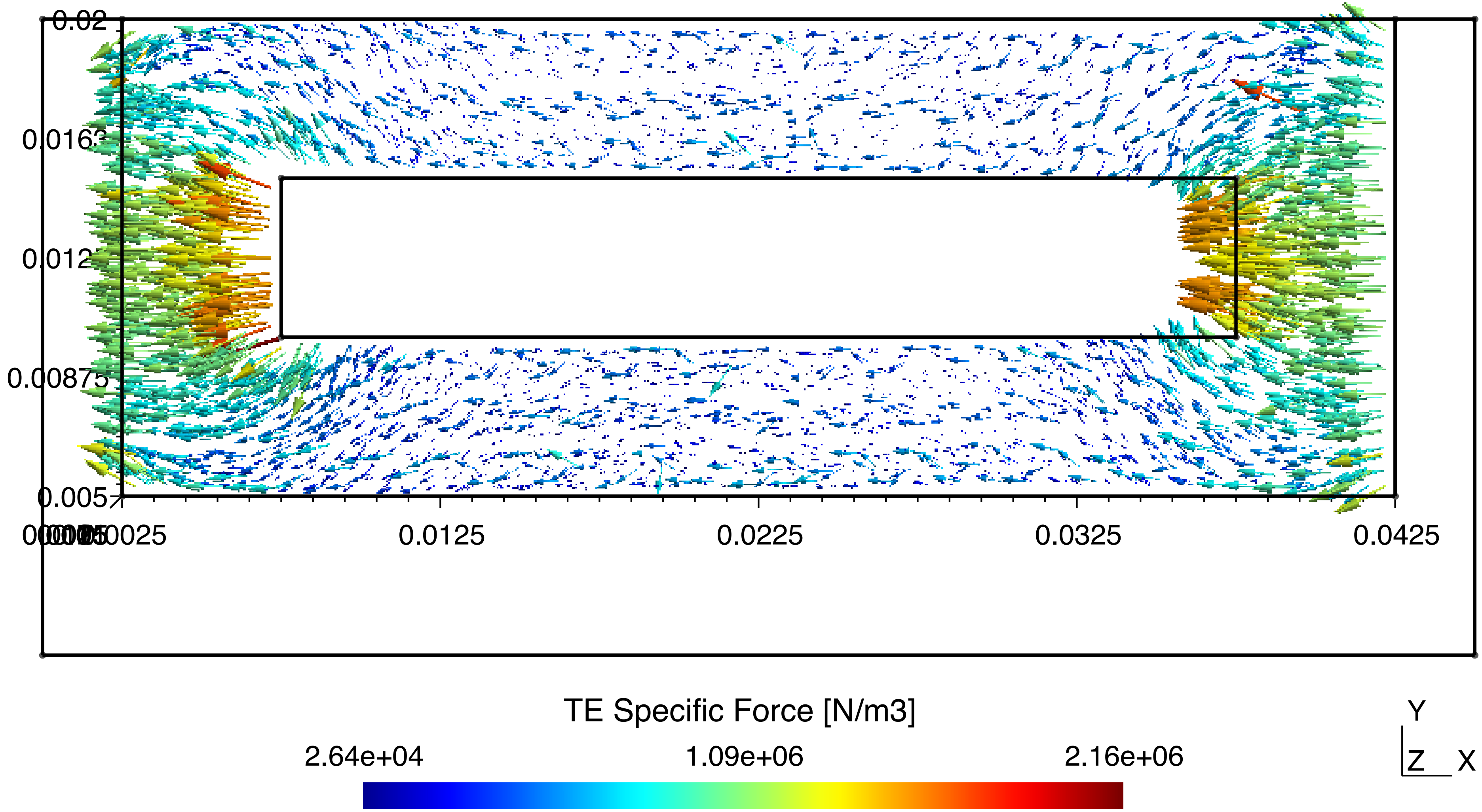
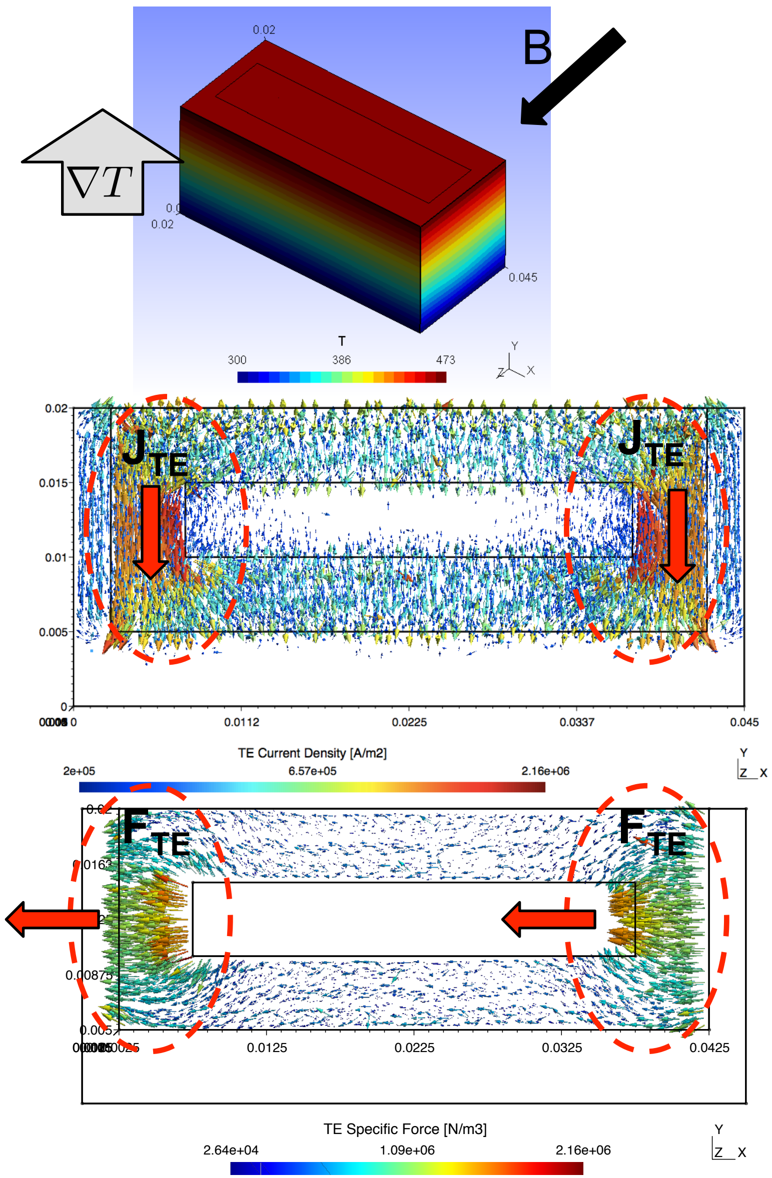
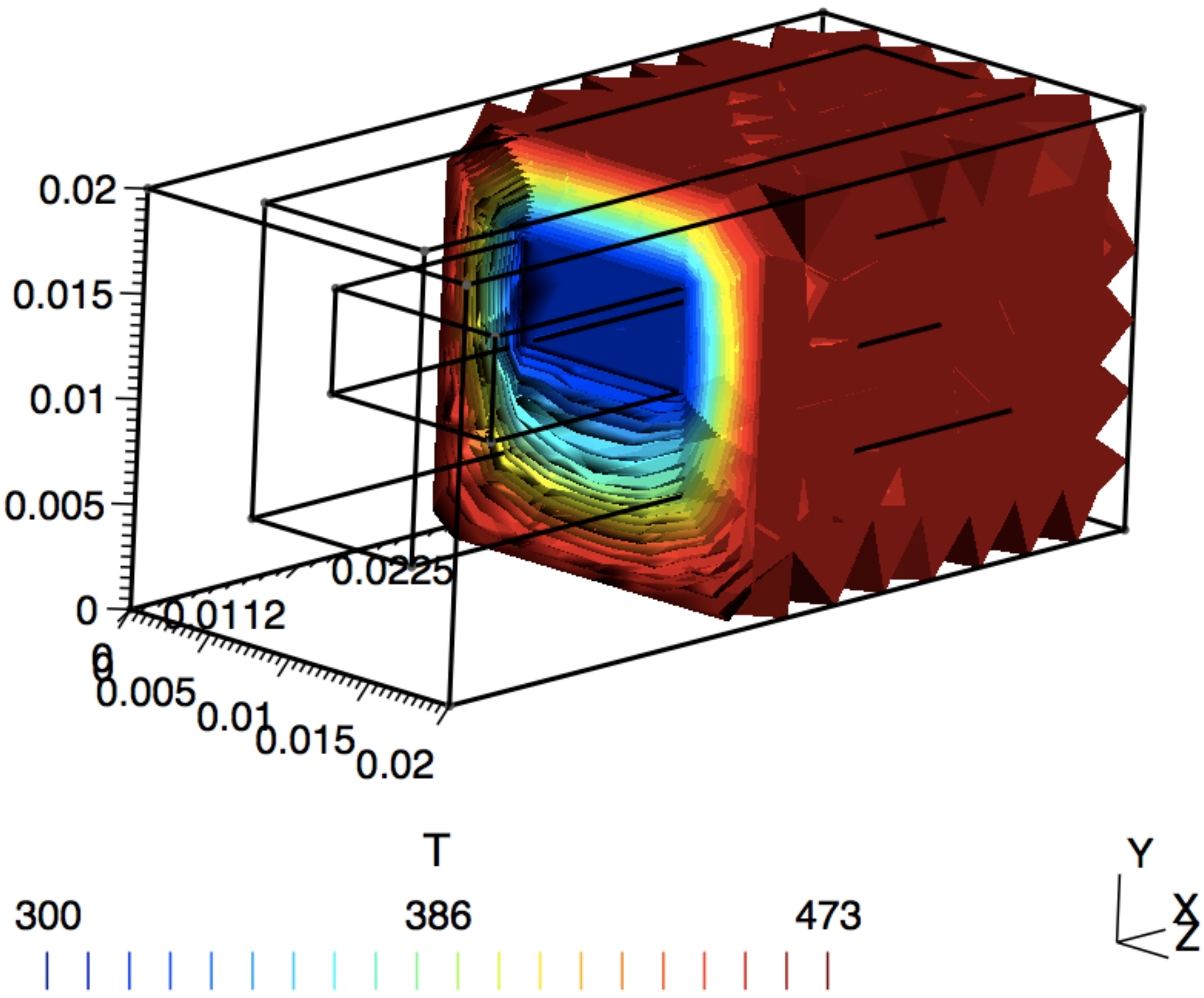


HEATING SCENARIO #1

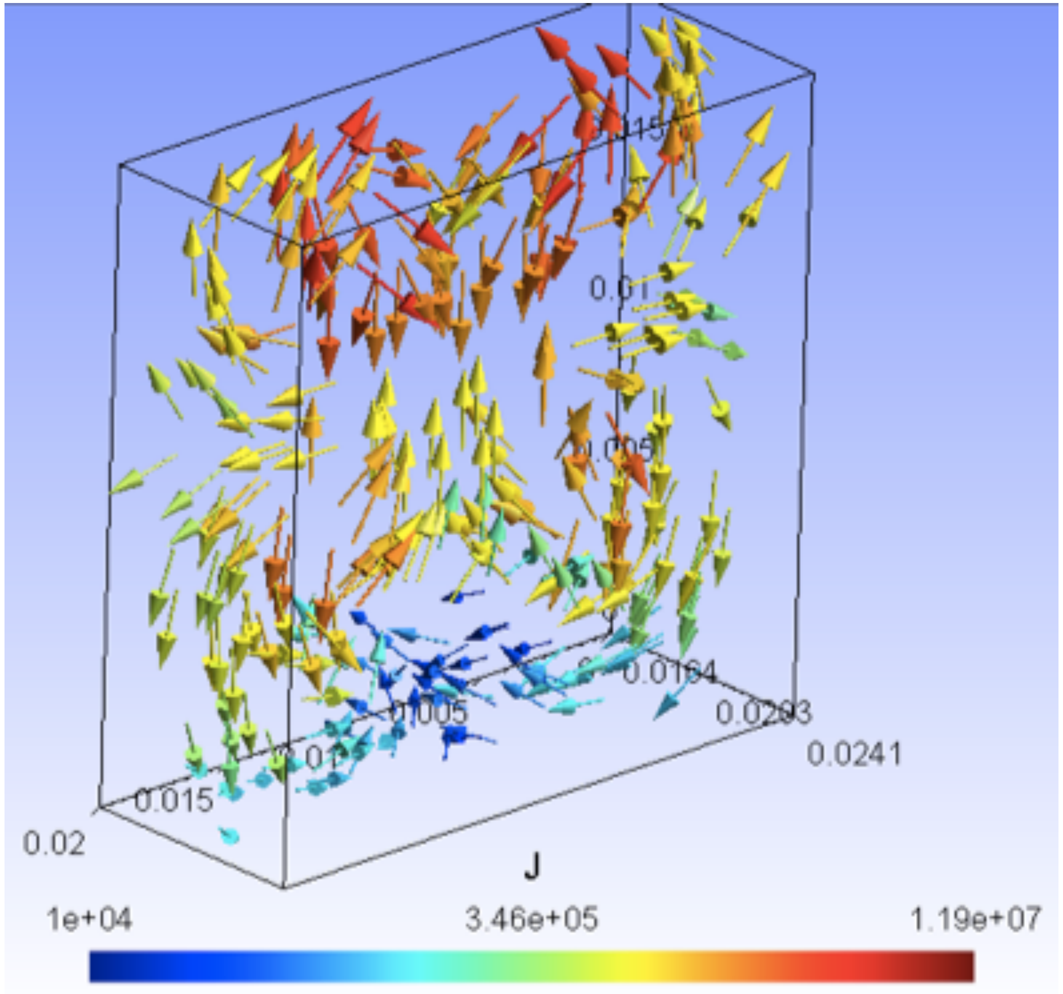
- With **heating on top and cooling on bottom**, a constant vertical gradient of temperature is developed through all the device (<u>pointing upward</u>)
- The **TE current density** is developed mainly in vertical direction (<u>pointing downward</u>), and is stronger at the two locations of the dashed ellipses in the figure
- The TE specific force is directed mainly **longitudinally in one direction** of the trench; all the liquid lithium is pushed toward the same direction
- The highest force is developed at the two extremities (dashed circles); the force is small and badly developed along the two channels



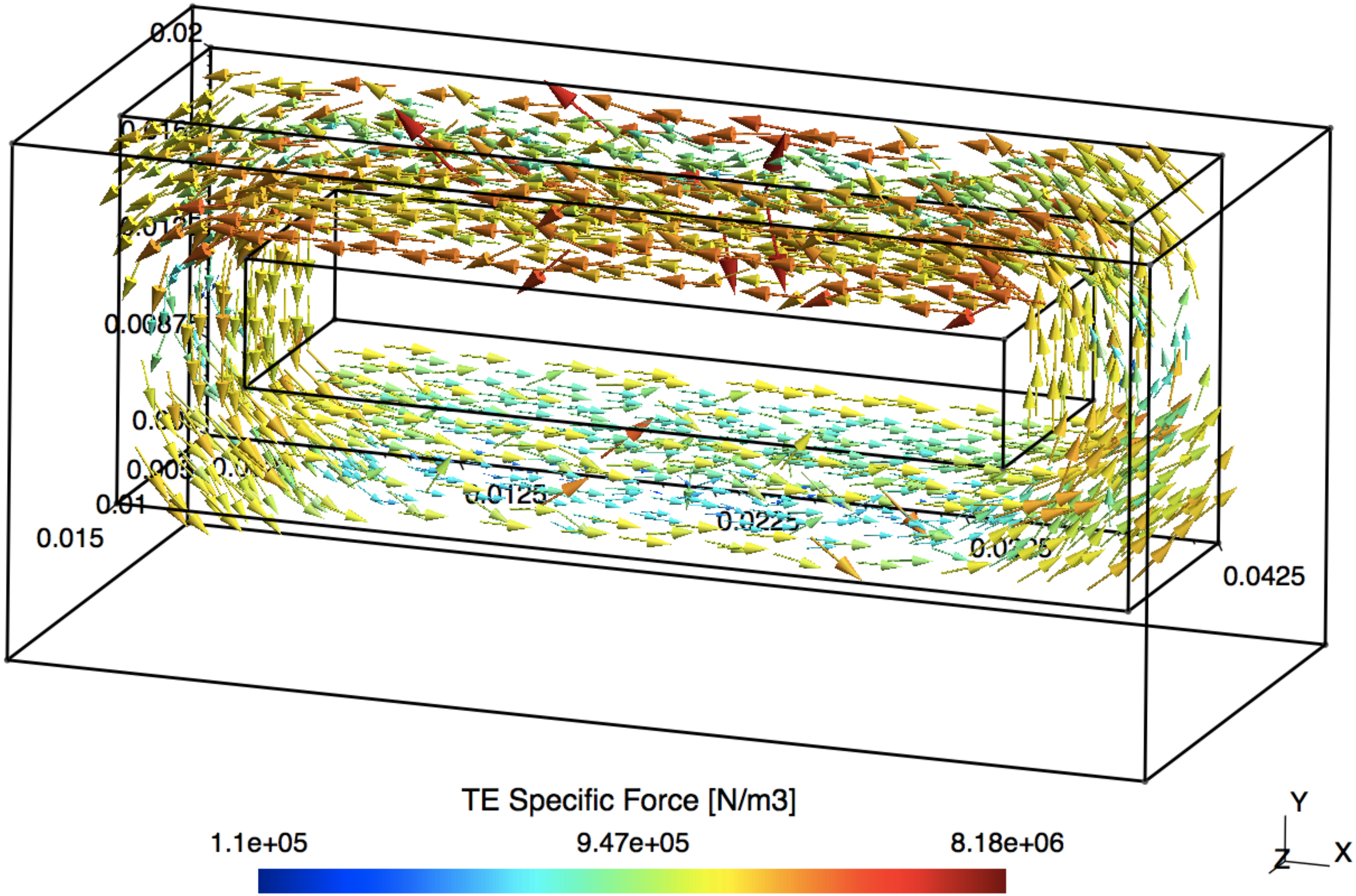
HEATING SCENARIO #2



HEATING SCENARIO #2



HEATING SCENARIO #2



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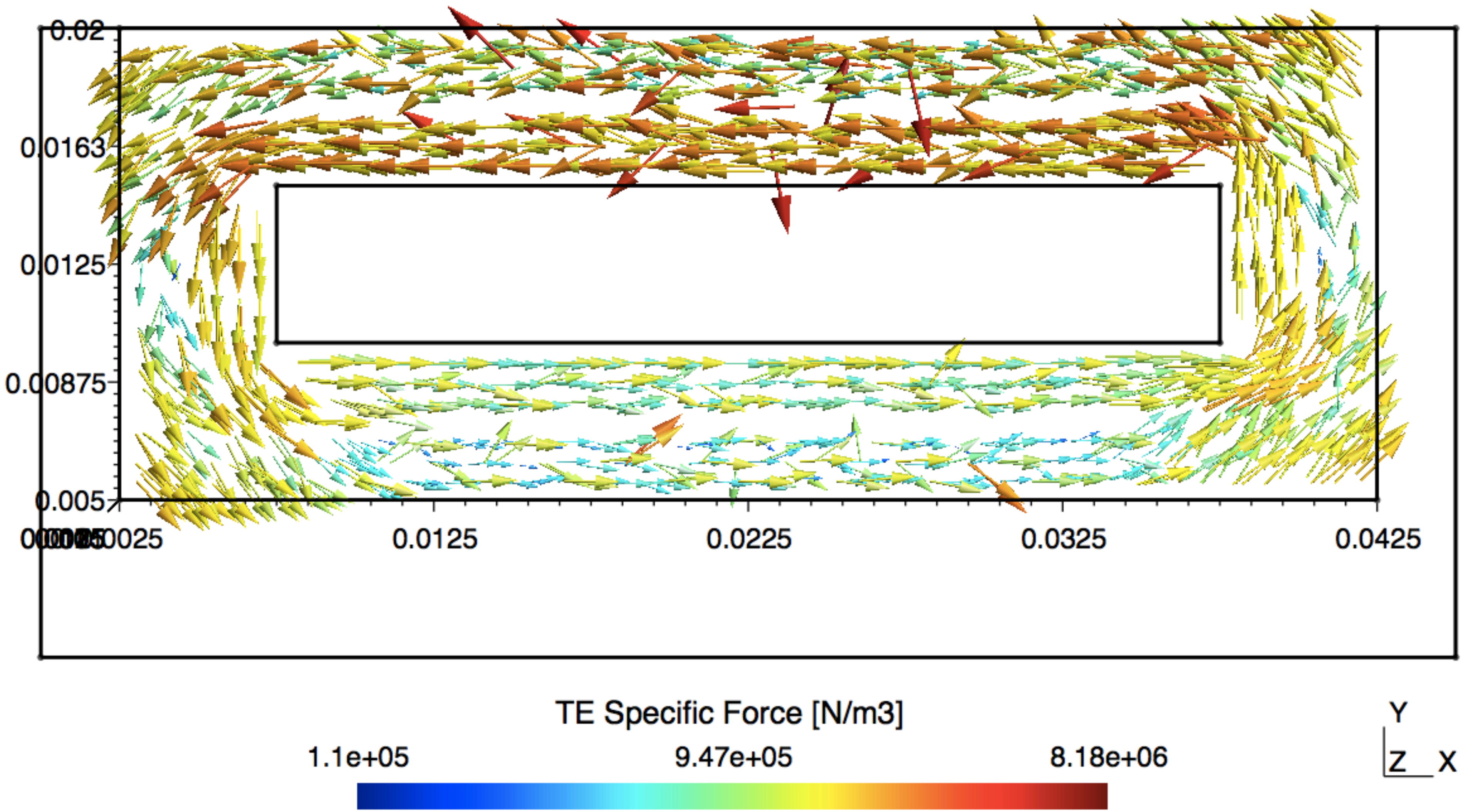
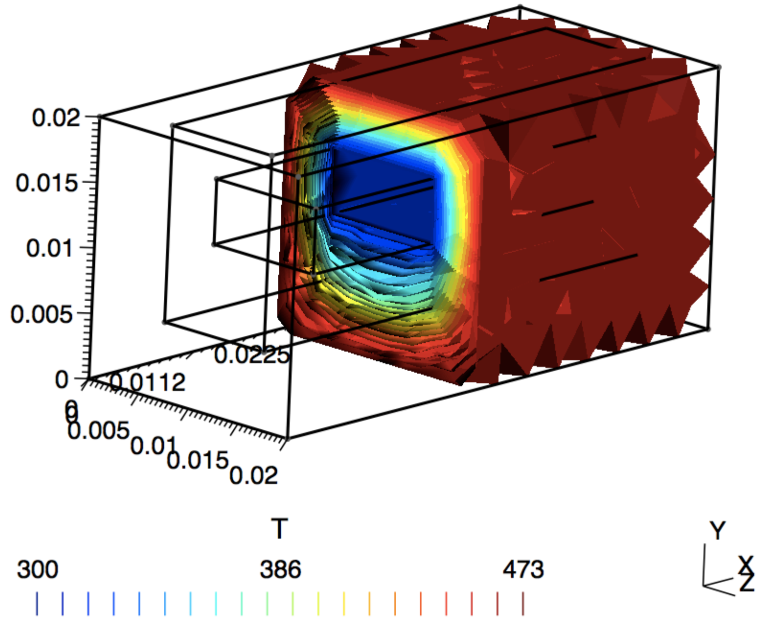
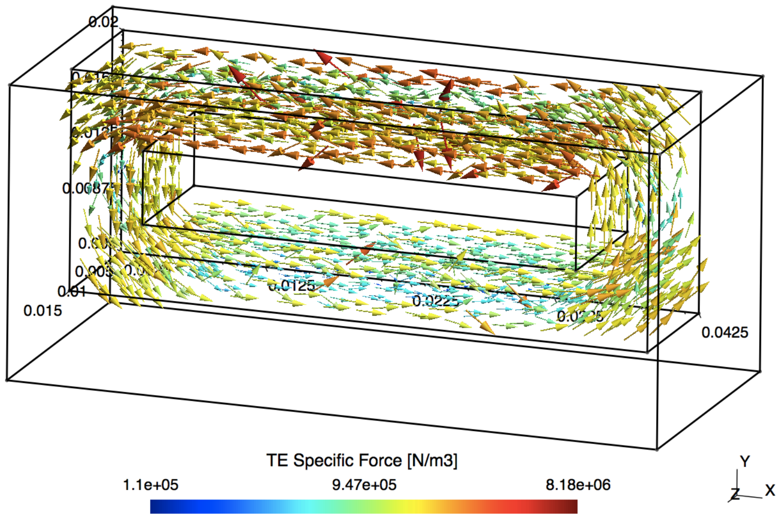


HEATING SCENARIO #2

- The picture changes when the **cooling is at center and the heating is from the sides**
- The top side is heated thanks to the thermal flux from the plasma; all the other sides are artificially heated with auxiliary heating elements
- The **temperature gradient** is directed radially outward, from the cooled center toward the heated walls
- After that the temperature gradient is established, the liquid lithium can be pumped through the channel by TEMHD forces only
- The **magnitude** of specific force is in the 10^5-10^6 N/m^3/T range, i.e. 1-2 orders of magnitude more than specific gravitational force, $f_g \sim \rho g$ ~ 508 kg/m^3 9.8 m/s^2 ~ 5x10^3 N/m^3





Assessment of second order effects

$$J = -\sigma \nabla \phi - \sigma S \nabla T$$

Generalized Ohm

$$q = -k \nabla T + (ST)J$$

Fourier – Peltier Law

$$\nabla \cdot J = 0$$

Current Continuity

$$\nabla \cdot q - \sigma^{-1} J \cdot J = 0$$

Heat Balance

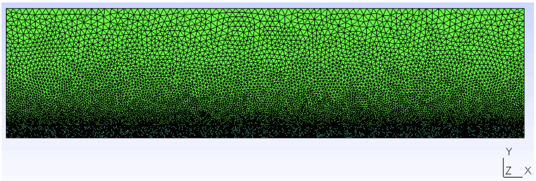
Assessment of second order effects

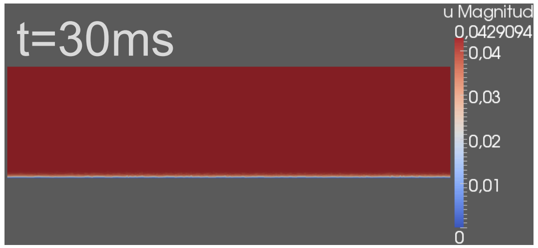
- Peltier effect
  - The introduction of the Peltier Heat Flux modifies the mathematical structure of the problem into a non-linear problem, solved by using Picard iteration and mixed finite elements.
  - Good convergence on the results, typically 7-8 iterations for RelTols 10^{-9}
- Joule heating
  - Joule heating gives an additional source of heat in the energy balance
  - The introduction of the Joule heating modifies the mathematical problem to a non-linear one, and iterative solutions must be implemented
  - Picard iterations require more iterations, typical convergence in ~20-30 iterations with RelTols 10^{-9}

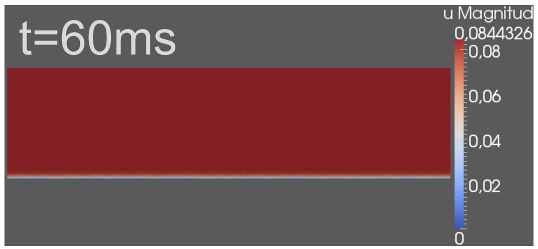
PROBLEM #2 : INCOMPRESSIBLE FORCED FLOW

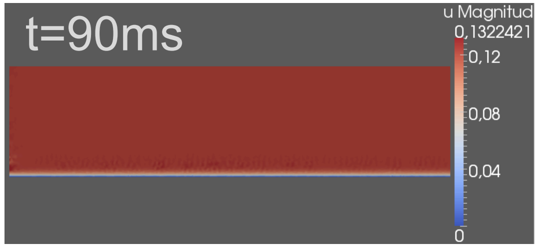
Incompressible NS forced by JxB TE force

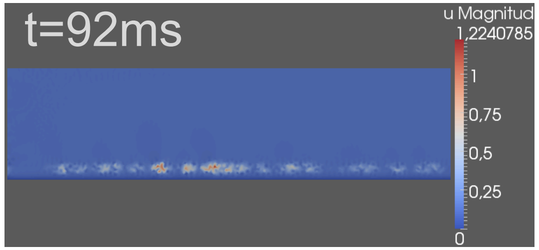
- Implementation of the classical projection method based on Helmholtz-Hodge decomposition (Chorin-Temam method,1968) for the solution of the incompressible NS plus the thermoelectric TE JxB force gives instability due to turbulence at the Boundary Layer

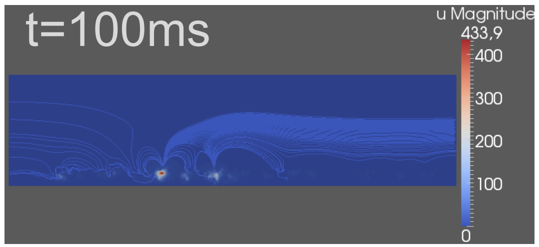


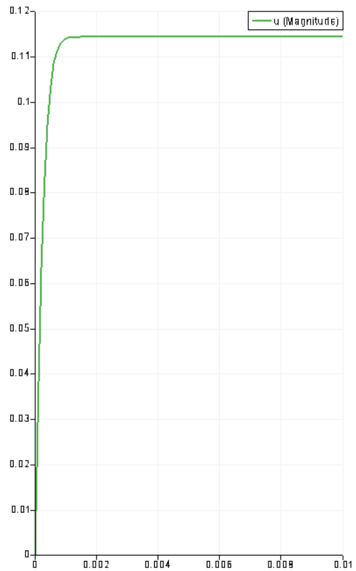


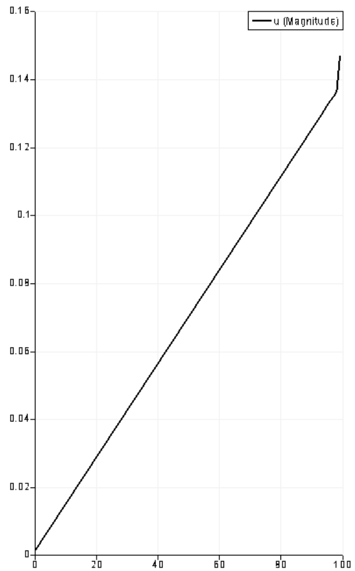




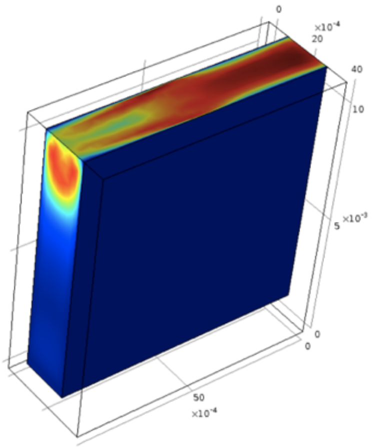








THE FULL PROBLEM



Lithium flow inside an elongated trench

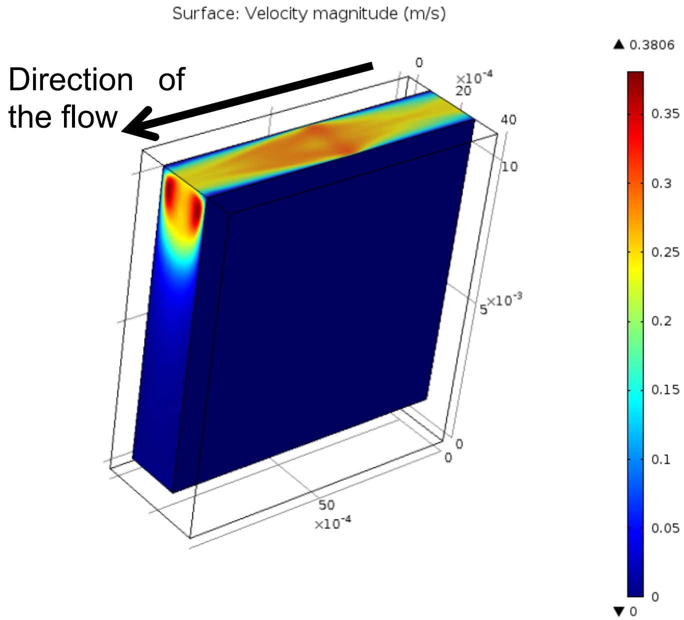
- COMSOL® allows to treat the basic physics of the TE-MHD problem almost completely (except 2nd order effects and Joule heating)

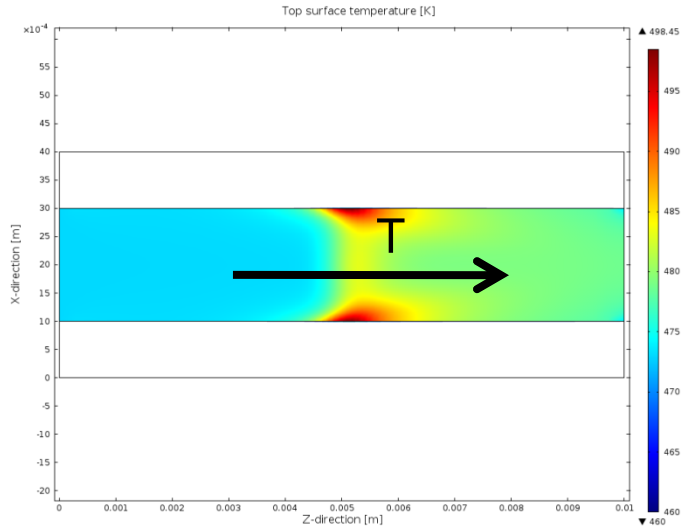
- Tests with the laminar solver exhibit a pattern with double velocity peaks near to the walls after the heated region; more tests with a classical k-epsilon treatment are under analysis

- Most of the high-velocity flow is developed close to the free surface at the top

- The peak flow velocity is of the order of tens of cm/s, thus allowing a practical removal of heat from the heated region

- At high magnetic fields the thickness of the Hartmann layer $\delta=1/B(\rho v/\sigma)^{1/2}$ becomes small and computations exhibit mesh-related challenges





Conclusions

- By using TEMHD, liquid metals can be flown by JxB forces in the high B-field environment of a tokamak
- Temperature gradients decide the direction and magnitude of the force, so they must be known and carefully controlled
- The electrical boundary conditions affect the current path, and all influencing parameters have to be accounted for:
  - location of the grounding
  - charging of nearby materials
  - fluxes of electric charges from plasma
  - Sharing of electrical currents between the plasma and the wall

References

[1] V. Surla at al., Journal of Nuclear Materials, 415, 18-22 (2011)

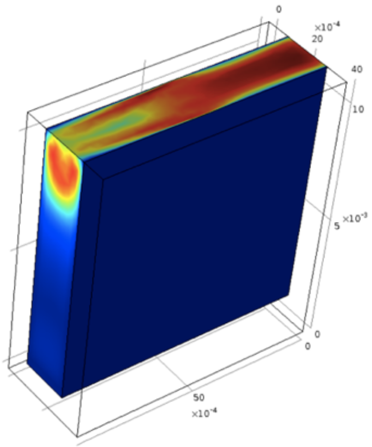
[2] J. A. Shercliff, J. Fluid Mech. 91, 2, 231-251 (1979)

[3] M. A. Jaworski et al., PRL 104, 094503 (2010)

[4] N. B. Morley, PhD dissrtation (1994)

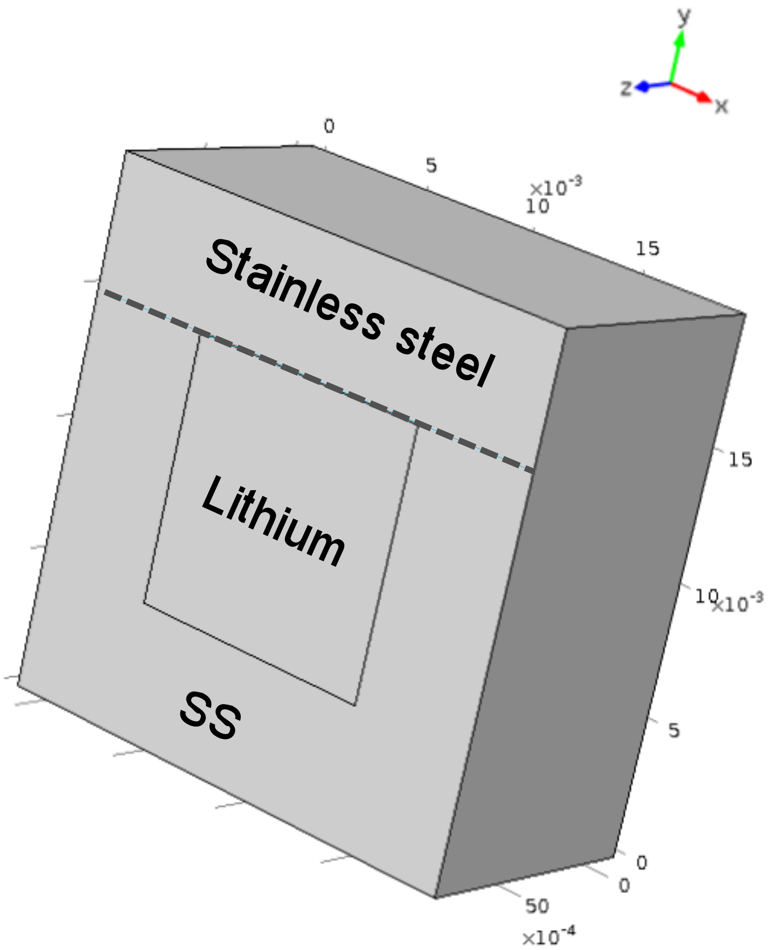
[5] S. Smolentsev at al., Fusion Sci. and Technology, Vol. 50, July 2006

APPENDIX A
MORE ON THE FULL PROBLEM



TEMHD flow in an infinite duct

- A closed duct of square sectionwas used to simulate an infinite long lithium flow in a stainless steel duct
- The lithium fluid is in a cube with 1.0cm of side
- The stainless steel is a 2.0 x 2.0 x 1.0 cm shell structure with 1.0 cm wall thickness
- The numerical convergence of the problem strongly depends on the Reynolds and Hartmann numbers
- At high magnetic fields the thickness of the Hartmann layer $\delta=1/B(\rho v/\sigma)^{1/2}$ becomes small and computations exhibit mesh-related challenges



TEMHD flow in an infinite duct

- Continuity

$$\nabla \cdot \vec{u} = 0$$

- Navier-Stokes

$$\rho\vec{u} \cdot \nabla\vec{u} = -\nabla P + \mu\nabla^2\vec{u} + \vec{J} \times \vec{B}$$

$$Periodic\ boundary\ condition\ for\ inlet\ and\ outlet$$

- Heat transfer

$$\rho C_P\vec{u} \cdot \nabla T = \nabla \cdot (k\nabla T)$$

$$Boundary\ condition:\ T|_{y=0} = 473K\ , T|_{y=0.02m} = 573K$$

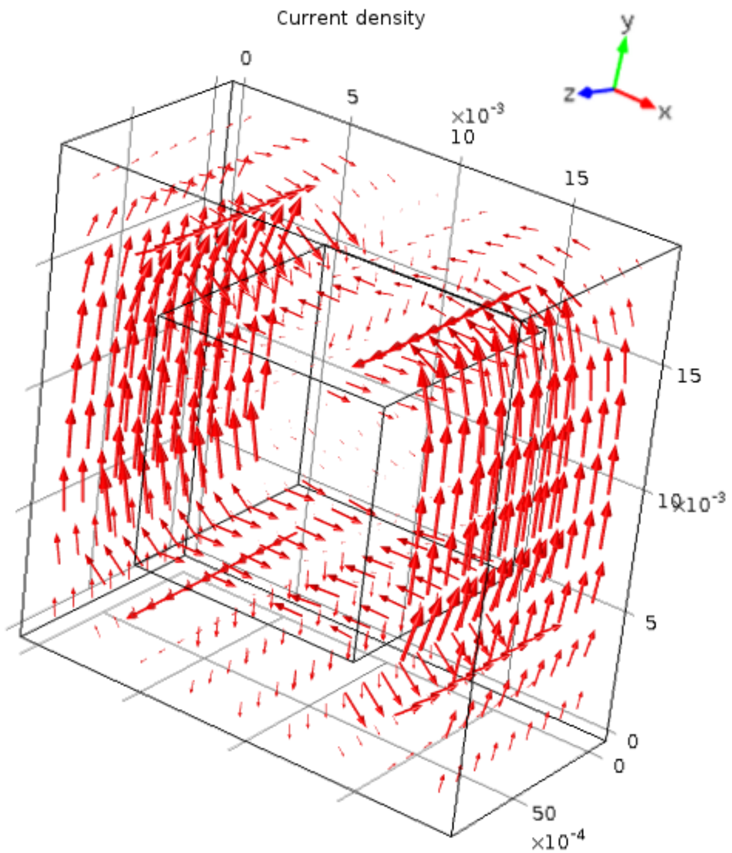
- Current conservation

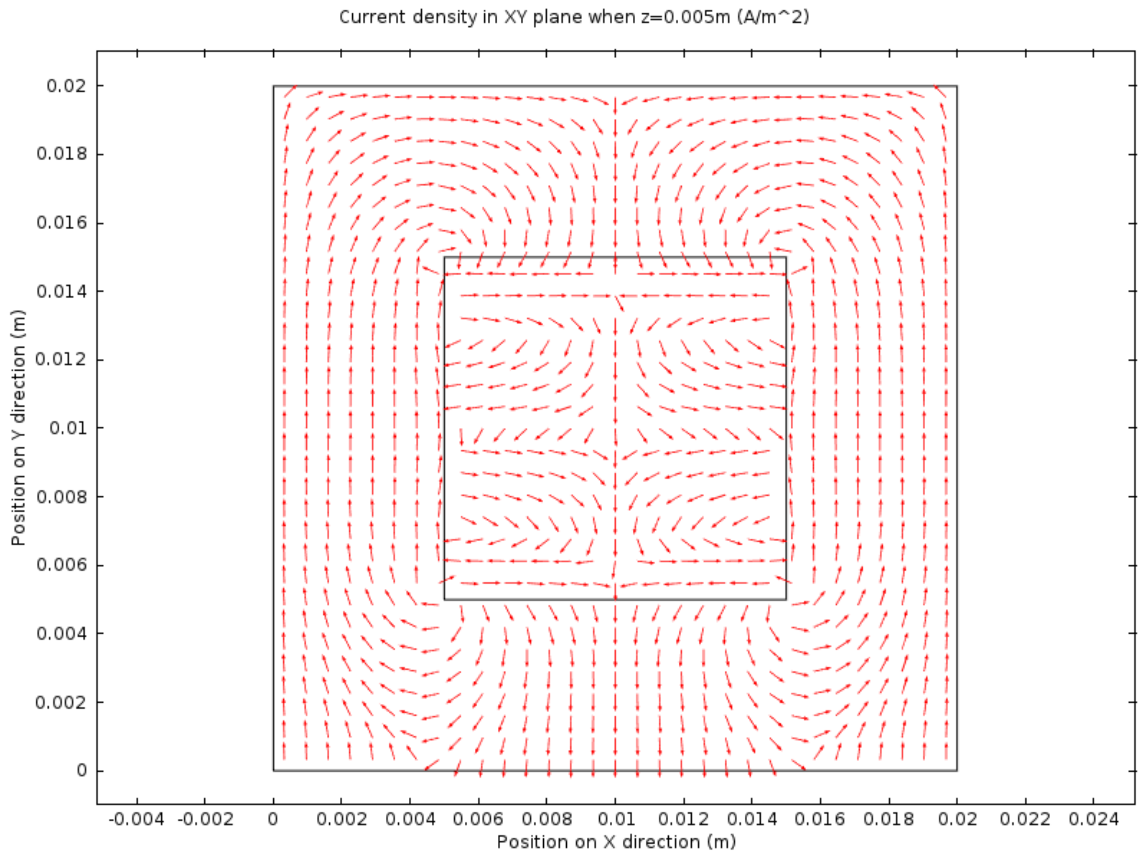
$$\nabla \cdot \vec{J} = 0\ and\ \vec{J} = \sigma\left(-\nabla\varphi + \vec{u} \times \vec{B} - S\nabla T\right)$$

$$Boudary\ condition: y = 0\ surface\ grounded, other\ surfaces\ insulated$$

- In above equations B is assumed to be constant. S is the Seebeck coefficient of lithium.

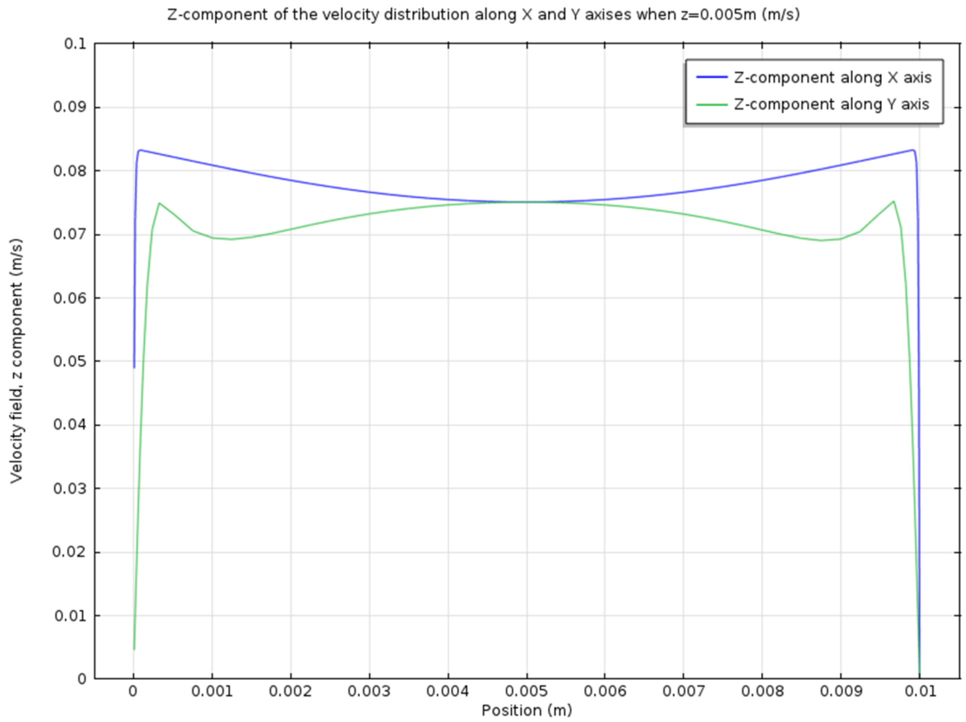
Current density

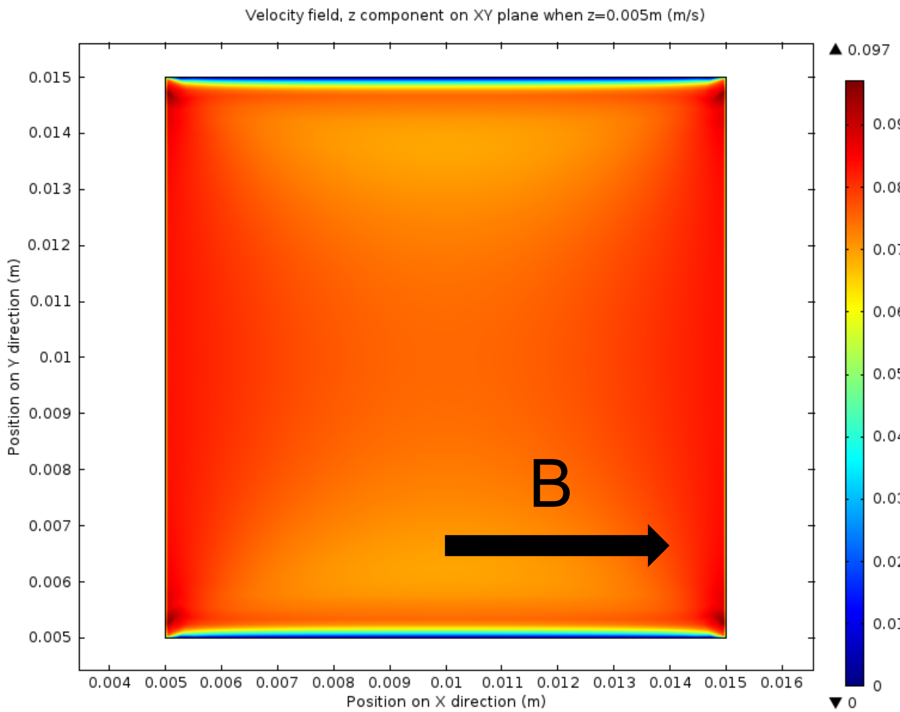


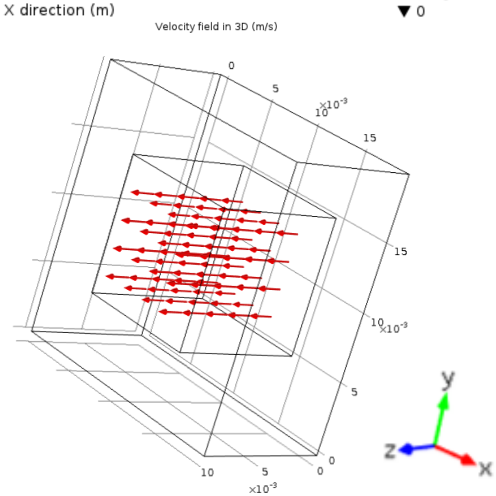


- The current pattern is smaller inside the lithium part because the thermoelectric current is cancelled by MHD current.
- Most of the current flow in XY plane accordingly to the temperature gradient.

Velocity field







- The velocity along B exhibits the classical profile from Hartmann layer
- The $S\nabla T$ term modifies the classical “Hartmann” solution allowing a flow without pressure gradients forces

Lithium flow inside an elongated trench

- COMSOL® allows to treat the basic physics of the TE-MHD problem almost completely (except 2nd order effects and Joule heating)

- A source term that represents the Seebeck effect can be added into the Ohm equation

$$\nabla \cdot \vec{u} = 0 \qquad \nabla \cdot \vec{J} = 0$$

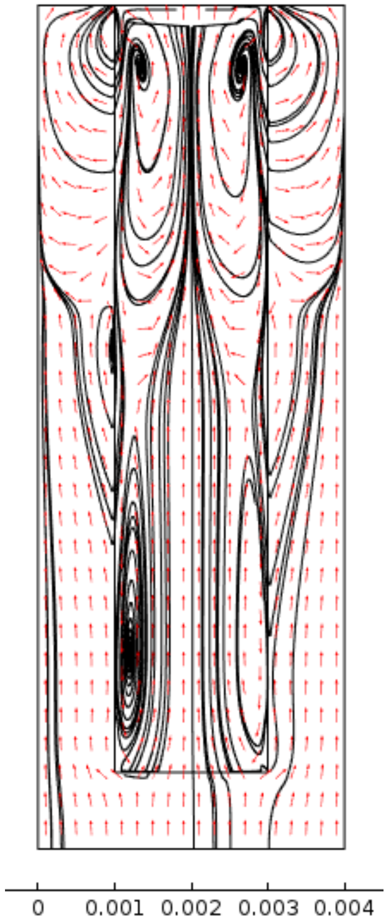
$$\vec{J} = \sigma(-\nabla\phi + \vec{u} \times \vec{B} - S\nabla T)$$

$$\rho \left[\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\,\vec{u} \right] = \nabla p - \mu \nabla^2 \vec{u} + \vec{J} \times \vec{B}$$

$$\rho C_p \left(\frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T \right) = \nabla \cdot (k \nabla T)$$

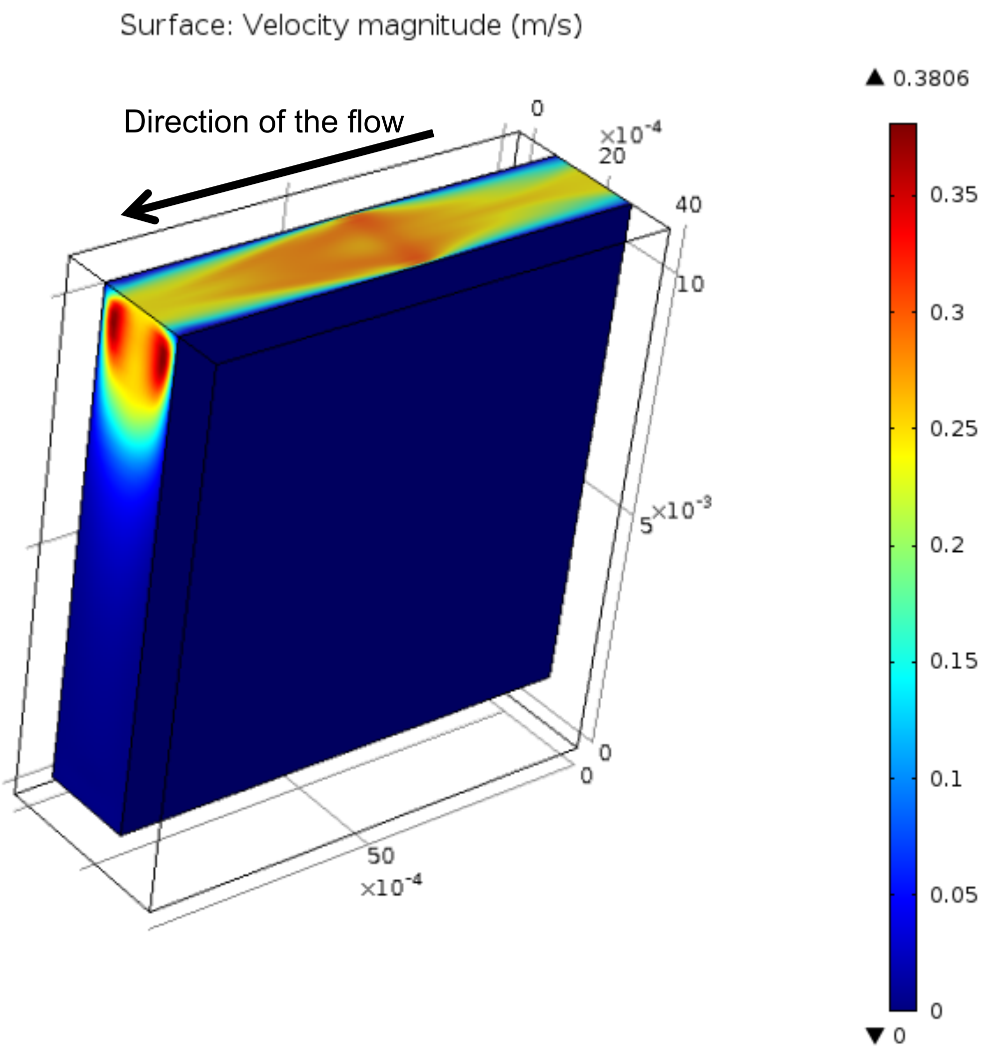
•COMSOL® simulations have been done by W. Xu

Streamlines of current density

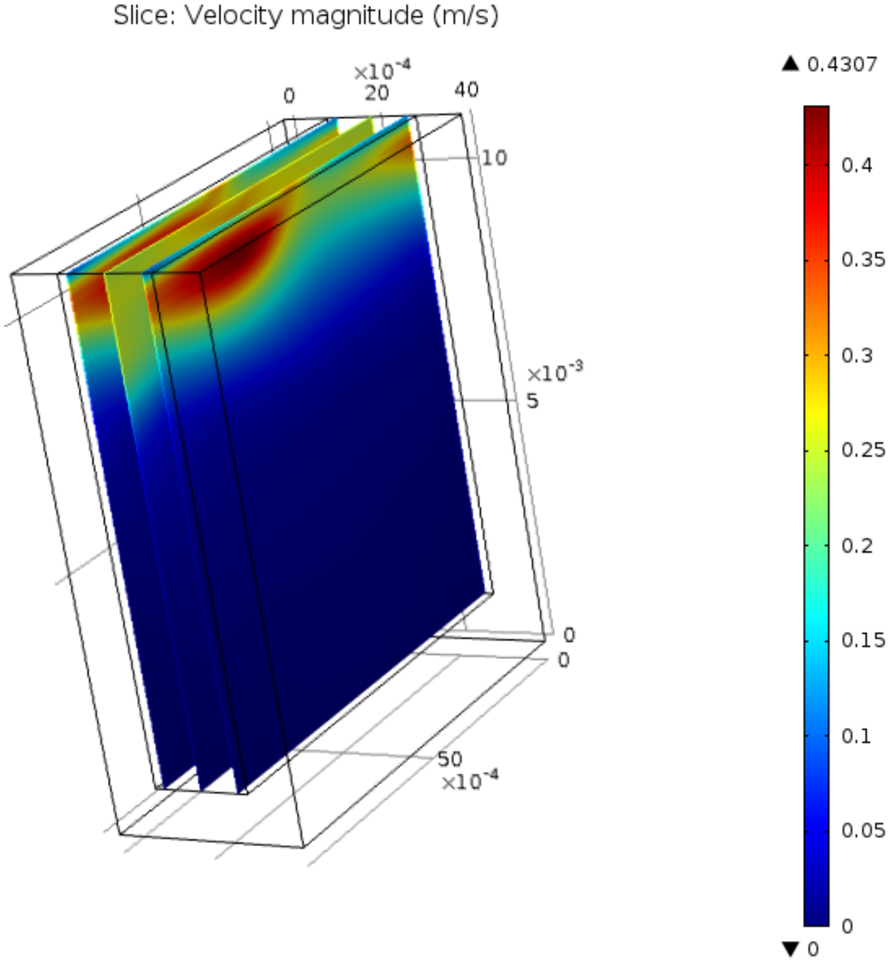


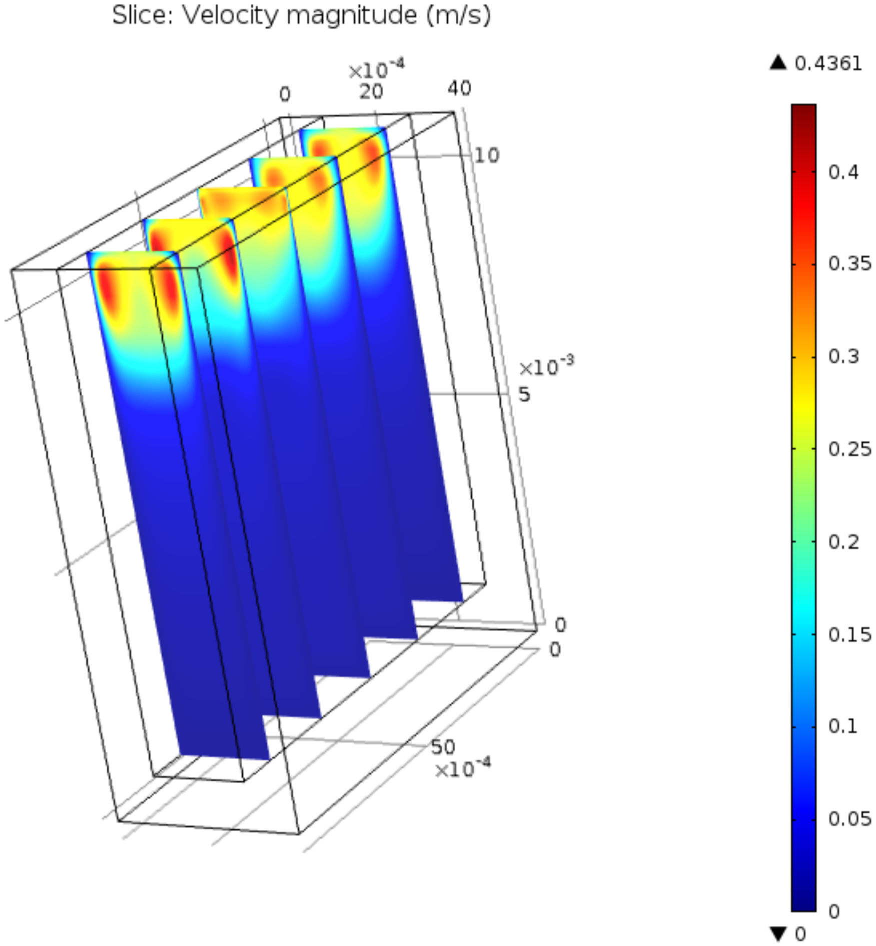
Lithium flow inside an elongated trench

- In stationary conditions a Lithium flow is developed along the trench, driven by the thermoelectric JxB force
- The flow exhibits a pattern with double velocity peaks near to the walls after the heated region
- Most of the high-velocity flow is developed close to the free surface at the top
- The peak flow velocity is of the order of tens of cm/s, thus allowing a practical removal of heat from the heated region

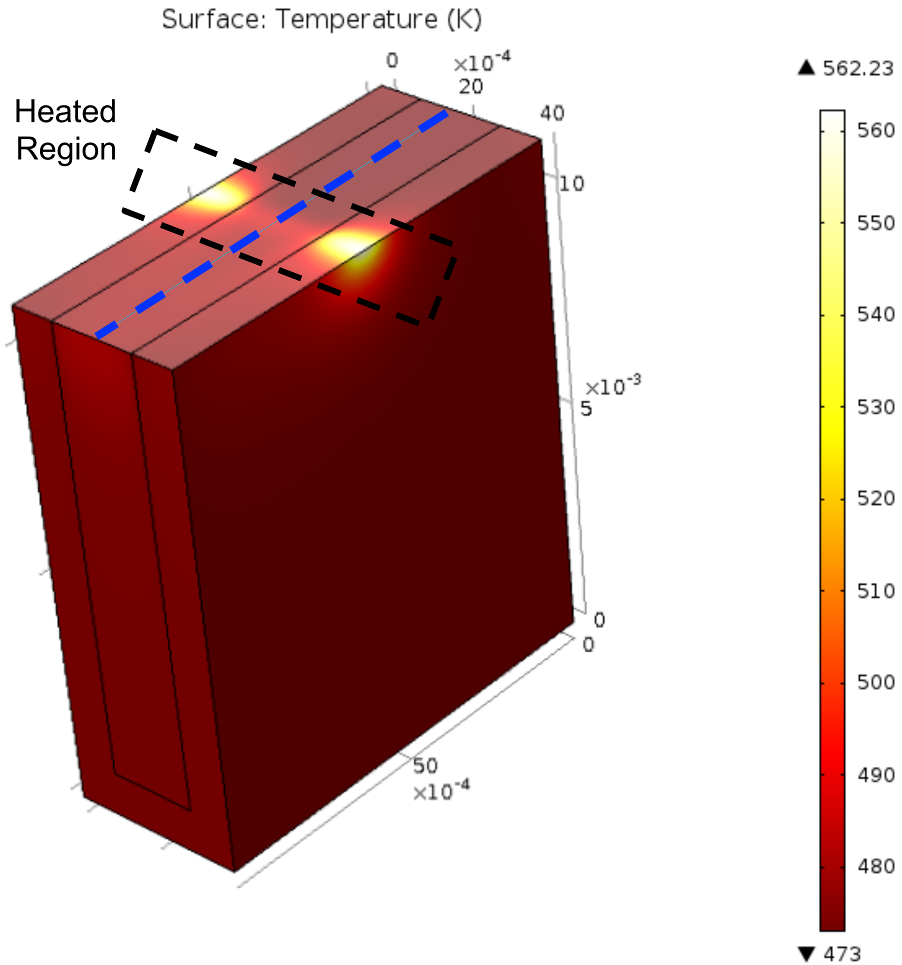


Features of the velocity field

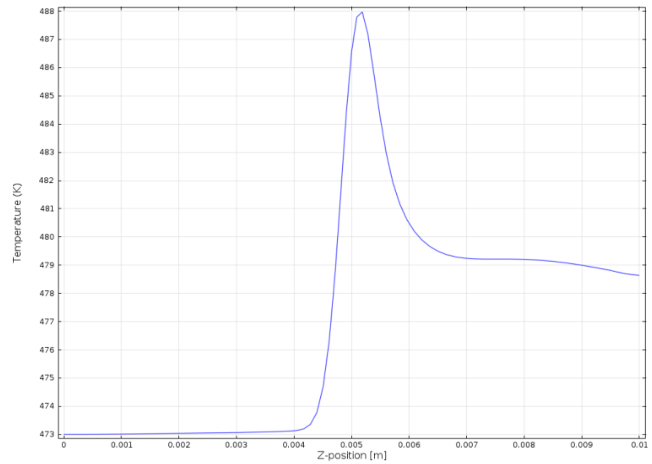


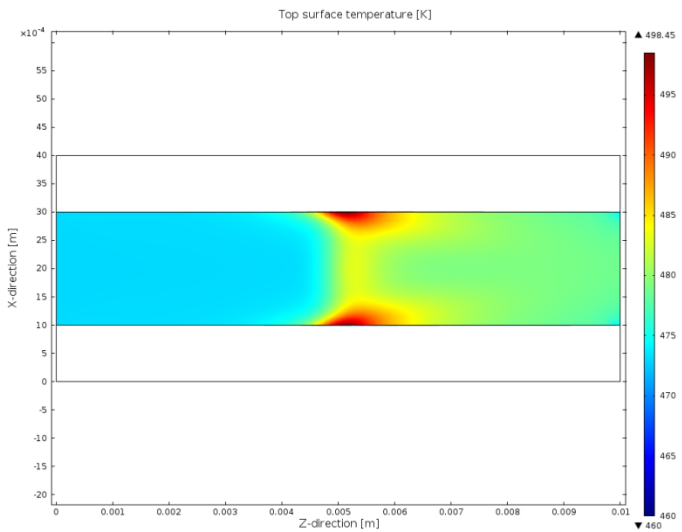


Temperature field and heat removal



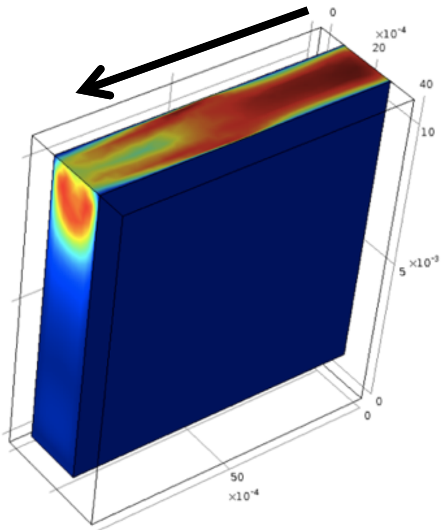
Temperature along the center line
(along the blue line in the 3D picture)

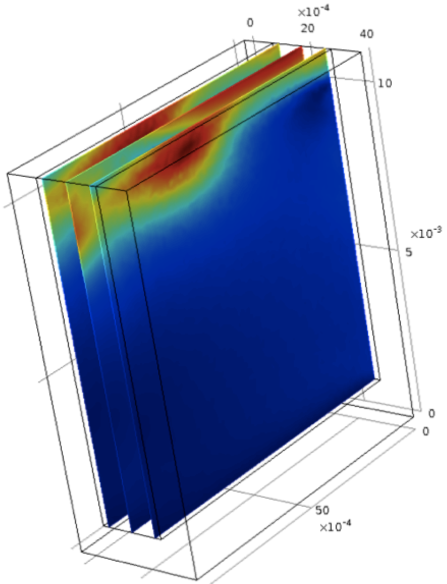


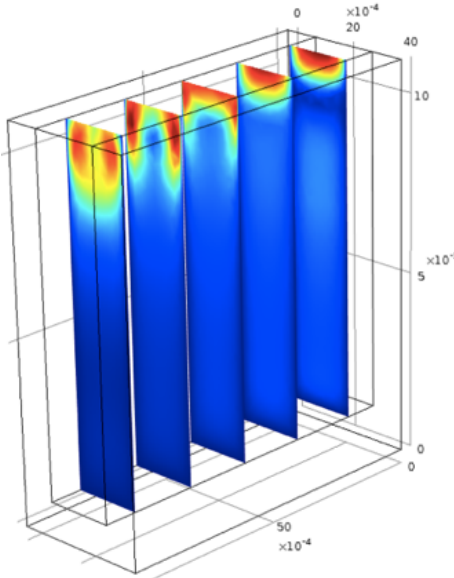


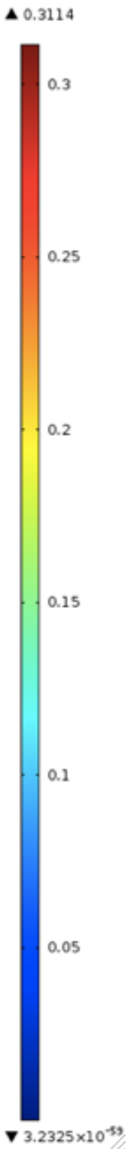
k-epsilon treatment of Li turbulence

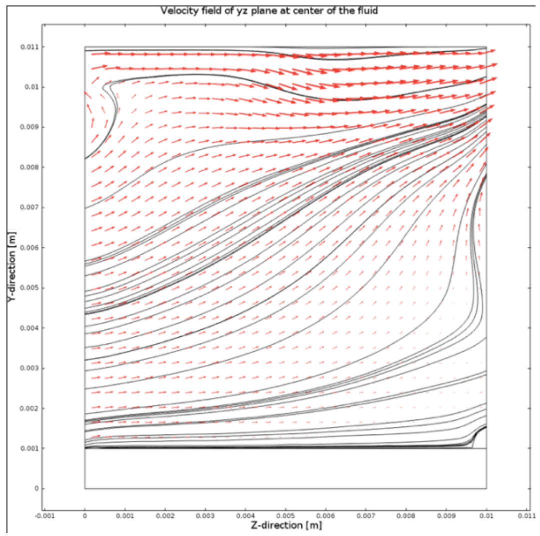
Preliminary 3D COMSOL® simulations using the standard k-ε turbulence model reveal features similar to the laminar case

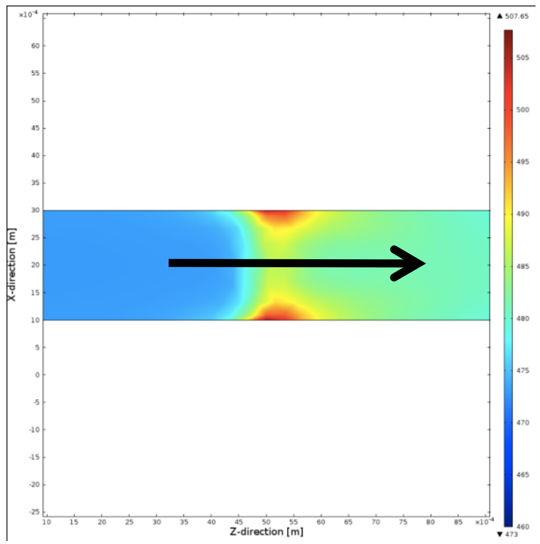




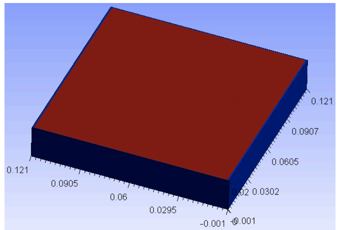








APPENDIX B
MORE ON THE SQUARE TRAY



Stainless steel tray filled with lithium

- In case of decentred heating
  - When the heating on top covers only partially the surface, a more complex current path of TE currents is developed
  - Most of the TE currents are generated only at the Li-SS interfaces interested by the strongest Grad-T
- Complex paths of thermoelectric forces are developed, depending on the local vectors of current density and magnetic field